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THE

AMERICAN

PHILOSOPHICAL ARITHMETIC

DESIGNED FOR THE USE OF ADVANCED CLASSES

IN

SCHOOLS AND ACADEMIES;

CONTAINING

THE ELEMENTARY AND THE MORE ADVANCED PRINCIPLES OF THE SCIENCE OF NUMBERS, AND THEIR APPLICATIONS TO PRACTICAL PURPOSES,

TOGETHER

WITH CONCISE AND ANALYTIC METHODS OF SOLUTION, AND ABBREVIATED METHODS OF COMPUTATION.

BY

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PREFACE.

I HAVE attempted in this work to present CLEARLY and CONCISELY, all those important *principles* and *properties* of numbers, which are necessary to a full comprehension of the higher branches of Mathematics, and their application to practical business and scientific calculations. The arrangement and illustration of these principles will enable the pupil to think for himself, independent of arbitrary rules, and summon to his aid on the instant all his arithmetical knowledge. This work treats extensively of mercantile transactions, interest, trade and barter, abbreviated arithmetical calculations, &c., and contains several principles not heretofore published, of extensive and practical application.

A Key containing additional methods of analysis, and answers, (if deemed indispensable,) will be prepared for the use of Teachers.

J. F. S.

UNIVERSITY OF NORTHERN PENNSYLVANIA, *July 4th*, 1853.

STODDARD'S

PHILOSOPHICAL ARITHMETIC.

CHAPTER I.

INTRODUCTORY DEFINITIONS.—NOTATION.—NUMERATION.

ARTICLE **1.** SCIENCE is knowledge systematized; that is, knowledge so classified and arranged as to be conveniently taught, easily acquired and remembered, readily referred to, and advantageously applied.

ART. **2.** ART is a judicious application of science to practical purposes.

ART. **3.** A UNIT, or *Unity*, is a single thing; as, a tree, an apple, a boy, &c.

ART. **4.** A NUMBER is either a unit or composed of an assemblage of units. One, two, three, &c., are numbers.

ART. **5.** QUANTITY is anything that will admit of measurement. A *line*, a *surface*, *time*, and other things of this nature, are quantities: but imagination, reason, virtue, &c., are not quantities, therefore, they are not subjects of mathematical investigation.

In common language, quantity or *numbers* are expressed by the words *one*, *two*, *three*, *four*, &c.; in arithmetical operations, by characters called *Figures*.

ART. **6.** MATHEMATICS is the science that treats of the properties and relations of quantity. Its fundamental branches are Arithmetic, Algebra, and Geometry.

ART. **7.** ARITHMETIC is the science of numbers, and the art of computation by them. It treats theoretically and practically of the nature and properties of numbers as employed in calculation.

ART. **8.** ALGEBRA is a general method of investigating the relations of quantity, by means of *letters* and *signs*, or *symbols*.

ART **9.** GEOMETRY is the science of magnitude ; it estimates and compares extension and form.

ART. **10.** AN ABSTRACT number is one that does not refer to any particular denomination ; as, *one*, *six*, *ten*, *five hundred*, &c.

ART. **11.** A CONCRETE, or *denominate* number is one that refers to some particular thing or denomination; as, *four* apples, *ten* dollars, &c.

SIMILAR *concrete* numbers express the *same kind* of units; as, *ten* dollars and *fifty* dollars.

DISSIMILAR *concrete* numbers express *different kinds* of units ; as *five* dollars, *ten* horses.

NOTATION.

ART. **12.** NOTATION is the art of expressing numbers by *figures*, *letters*, or other symbols.

The Romans used the seven following letters to express numbers, which we now use to number Lessons, Chapters, &c. :—I, for one ; V, for five ; X, for ten ; L, for fifty; C, for one hundred ; D, for five hundred ; M, for one thousand. The intermediate numbers and numbers greater than a thousand are expressed by repetitions and combinations of these letters, as exhibited in the following

ROMAN TABLE.

One is represented by		I.	
Two	" "	II.	
Three	" "	III.	As often as a letter is repeated, its value is repeated. Thus : X, is ten , XX, twenty, &c.
Four	" "	IV.	
Five	" "	V.	
Six	" "	VI.	
Seven	" "	VII.	
Eight	" "	VIII.	
Nine	" "	IX.	

Ten is represented by X.
Eleven " XI.
Twelve " XII.
Thirteen " XIII.
Fourteen " XIV.
Fifteen " XV.
Sixteen " XVI.
Seventeen " XVII.
Eighteen " XVIII.
Nineteen " XIX.
Twenty " XX.
Thirty " XXX.
Forty " XL.
Fifty " L.
Sixty " LX.
Seventy " LXX.
Eighty " LXXX.
Ninety " XC.
One hundred " C.
Five hundred " D.
One thousand " M.
Five thousand " $\overline{\text{V}}$.

A letter of less value placed before one of greater, diminishes its value as much as the value of the letter placed there; if placed after the same letter it increases its value by the same number. Thus, if before X, ten, we place I, one, it becomes IX, nine; if after, it becomes XI, eleven. If before L, fifty, we place X, ten, it becomes XL, forty; if after, it becomee LX, sixty, &c.

A bar (—) placed over any letter increases its value a thousand fold. Thus, $\overline{\text{IV}}$ is four thousand.

ARABIC NOTATION.

ART. **13.** In all arithmetical calculations numbers are expressed by the *Arabic system of Notation.* This system of notation employs the following ten characters, called *Figures :*

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	cipher, zero, or naught.

Hence, *figures* are representatives of numbers, or *expressions* of *quantity.*

ART. **14.** The first nine of the above characters, are called *significant* figures, as each one represents a definite number when standing alone, while the last has *no value,* and is, therefore, of itself insignificant.

ART. **15.** The *significant* figures are also called *Digits*, from the Latin *digitus*, a finger, because, many centuries ago, people used to do their reckoning by counting their fingers. The use of the ten fingers is supposed to have originally suggested the idea of employing ten characters to express numbers.

ART. **16.** Notwithstanding the 0, *cipher*, has no value of itself, yet it is of as much importance as either of the *digits*, as it serves in a peculiar manner to change their value by changing their *locality;* hence, by some it has been called a *Locater*.

NUMERATION.

SIMPLE AND LOCAL VALUES OF FIGURES.

ART. **17.** A figure standing alone, or occupying the first place on the right of a row of figures, expressing a whole number, is denominated *units ;* that occupying the second place, *tens ;* that occupying the third place, *hundreds*, &c.

Thus,

Hds.	Tens.	Units.
4	2	8

is read four hundreds, two tens, and eight units, or *four hundred* and *twenty-eight*. Nine is the largest number that can be expressed by a single figure ; hence, numbers greater than nine must be expressed by some combination of these numerals.

ART. **18.** The SIMPLE *value* of a figure is its value when occupying unit's place.

The name of a figure expresses the *simple value* of that figure. Each of the *nine* digits as referred to in Art. 13, has its *simple value.* In the number 35, the *five* has its simple value, *five*, and the *three* a *local* value.

ART. **19.** The LOCAL *value* of a figure is that which arises from its location. In the number 35, (above referred to,) the local value of the three is *three* tens, or *thirty* In the number 465, the local value of the 4 is

four hundreds, and that of the 6 is *six* tens, or *sixty*. The 5 has its simple value, *five*.

ART. **20.** When the nine digits occupy the second, or ten's place, each will then express the same number of tens that it did units, when occupying the place of units. When they occupy the third, or hundreds' place, each will then express as many hundreds as it did units when in the place of units.

NOTE.—This will be rendered plain by inspecting the following

TABLE.

							Tens. Units.
No	units	and	one	ten,	or	*ten,*	10
One	unit	"	"	"	"	*eleven,*	11
Two	units	"	"	"	"	*twelve,*	12
Three	"	"	"	"	"	*thirteen,*	13
Four	"	"	"	"	"	*fourteen,*	14
Five	"	"	"	"	"	*fifteen,*	15.
Six	"	"	"	"	"	*sixteen,*	16
Seven	"	"	"	"	"	*seventeen,*	17
Eight	"	"	"	"	"	*eighteen,*	18
Nine	"	"	"	"	"	*nineteen,*	19
No	"	"	two	"	"	*twenty,*	20

NOTE.—The terms thirteen, fourteen, fifteen, sixteen, &c., are obviously contractions of three and ten, four and ten, five and ten, six and ten, &c.

In a similar way, by contracting the expressions two tens, three tens, four tens, &c., the expressions twenty, thirty, forty, &c., are derived.

Twenty-one, Twenty-two, Twenty-three, Twenty-four, Twenty-five, Twenty-six, Twenty-seven, Twenty-eight, and Twenty-nine, are respectively expressed by placing in regular order the digits in the place of the cipher in the number 20. In a similar manner, numbers from Thirty to Forty, from Forty to Fifty, from Fifty to Sixty, &c , are expressed by placing the digits in the place of the cipher in the numbers Thirty, Forty, Fifty, Sixty, &c.

							Tens. Units.
No	units	and	three	tens,	or	*Thirty,*	30
"	"	"	four	"	"	*Forty,*	40
"	"	"	five	"	"	*Fifty,*	50
"	"	"	six	"	"	*Sixty,*	60
"	"	"	seven	"	"	*Seventy,*	70
"	"	"	eight	"	"	*Eignty,*	80
"	"	"	nine	"	"	*Ninety,*	90

The terms twenty-one, twenty-two, &c., are compounded of *twenty* and *one*, *twenty* and *two*, &c. Other numbers expressed by two figures are similarly formed.

One	hundred,	100
Two	"	200
Three	"	300
Four	"	400
Five	"	500
Six	"	600
Seven	"	700
Eight	"	800
Nine,	"	900

ART. **21.** By inspecting the above table, it will be observed, that a figure standing in the second place, or place of tens, is *ten* times as great as though it were in the first or units' place. A figure that stands in the third place, or place of hundreds, is *ten* times as great as though it were in tens' place, and one hundred times as great as though it were in the place of units. Hence, ten units make one ten, and ten tens make one hundred. We therefore infer universally, that

ART. **22.** *Figures increase in value from right to left in a ten-fold ratio; that is, each removal of a figure one place towards the left increases its value ten times.*

ART. **23.** As figures in the Arabic system of notation increase in ten-fold ratio from right to left, and decrease in the same ratio in an opposite direction, it is called the *Decimal* system of notation. The word *decimal* is derived from the Latin, *decem*, ten.

ART. 24. NUMERATION is the art of reading numbers, expressed by figures.

NOTE.—By carefully studying the following Table, the pupil will soon be able to read any number which requires not more than *nine* figures to express it.

TABLE.

Figures occupying the place of units, are sometimes called *units* of the *first order*, —those occupying the place of tens, *units* of the *second order*,—those accupying the place of hundreds, *units* of the *third order*, &c., as shown by the Table.

To read these numbers, first denominate each figure by the names units, tens, &c., as shown by the *table*, and then read from left to right as follows :—

Hundreds of Millions, or units of the 9th order.	Tens of Millions, or units of the 8th order.	Millions, or units of the 7th order.	Hundreds of Thousands, or units of the 6th order.	Tens of Thousands, or units of the 5th order.	Thousands, or units of the 4th order	Hundreds, or units of the 3d order.	Tens, or units of the 2d order.	Units, or units of the 1st order.	
.	.	.	.	.	.	.	.	4	Four.
.	.	.	.	.	.	.	4	5	Forty-five.
.	.	.	.	.	.	4	5	3	Four hundred and fifty-three.
.	.	.	.	.	4	5	3	2	Four thousand five hundred and thirty-two
.	.	.	.	4	5	3	2	6	Forty-five thousand three hundred and twenty-six.
.	.	.	4	5	3	2	6	7	Four hundred fifty-three thousand two hundred and sixty-seven.
.	.	4	5	3	2	6	7	8	Four millions, five hundred thirty-two thousand, six hundred and seventy-eight.
.	4	5	3	2	6	7	8	1	Forty-five millions, three hundred twenty-six thousand, seven hundred and eighty-one
4	5	3	2	6	7	8	1	9	Four hundred fifty-three millions, two hundred sixty-seven thousand, eight hundred and nineteen.

REMARK.—It would be well to write the figures of this *Table* on the black board, and have the pupils read them individually as well as collectively.

This Table shows plainly the simple and local values of figures. Each figure, except those in the place of units, has a local value, which may be named by the pupil as the teacher points to them separately.

ART. **25.** In the United States and continental Europe the French method of numeration is in general use. In this method of numeration a different name is given to every three figuers, counting from the right.

The first period contains units, tens of units, hundreds of units, and is therefore called the period of *Units*. For a similar reason the next left hand period is called the eriod of *Thousands*, &c.

ART. **26.** In the following *Table* the words above the row of figures express the particular denomination of the figures over which they are placed.

To read a number expressed by figures :—

Denominate each figure from right to left, remembering the name of each period, then read the figures of each period, beginning at the left hand, in the same manner as those of the period of units are read, and at the end of each period give its name.

FRENCH METHOD OF NUMERATION.

Hunds. of Decillions.	7	Period of Decillions.
Tens of Decillions.	1	
Decillions.	3,	
Hunds. of Nonillions.	8	Period of Nonillions.
Tens of Nonillions.	2	
Nonillions.	4,	
Hunds. of Octillions.	6	Period of Octillions.
Tens of Octillions.	2	
Octillions.	3,	
Hunds. of Septillions	8	Period of Septillions.
Tens of Septillions.	5	
Septillions.	7,	
Hunds. of Sextillions.	8	Period of Sextillions.
Tens of Sextillions.	6	
Sextillions.	2,	
Hunds. of Quintillions.	0	Period of Quintillions.
Tens of Quintillions.	7	
Quintillions.	3,	
Hunds. of Quadrillons.	4	Period of Quadrillions.
Tens of Quadrillions.	1	
Quadrillions.	0,	
Hunds. of Trillions.	2	Period of Trillions.
Tens of Trillions.	0	
Trillions.	6,	
Hunds. of Billions.	3	Period of Billions.
Tens of Billions.	7	
Billions.	4,	
Hunds. of Millions.	3	Period of Millions.
Tens of Millions.	8	
Millions.	6,	
Hunds. of Thousands.	8	Period of Thousands.
Tens of Thousands.	4	
Thousands.	7,	
Hundreds.	8	Period of Units.
Tens	6	
Units	2.	

EXERCISES IN NUMERATION.

Read the following numbers :—

Ex.		Ex.		Ex.	
1.	125	6.	63245	11.	836478492
2.	286	7.	732123	12.	326489472
3.	397	8.	8324671	13.	8246721478
4.	8462	9.	1247632	14.	448624783146
5.	98623	10.	463246273	15.	12345678901234

EXERCISES IN NOTATION.

ART. **27.** To express numbers by figures .—

Begin at the left, and write the figures of the highest order mentioned, observing to place in each order, the figures belonging to it, and when no digit is mentioned, to fill the place with a cipher.

Express the following numbers by figures :—

1. Forty-three.
2. Eighty-nine.
3. Three hundred and eight.
4. Four thousand, one hundred and four.
5. Seventy-five thousand and seventy-five.
6. Six hundred and five thousand, one hundred and twenty-three.
7. Eight hundred and seventy-two thousand, five hundred and twelve.
8. Nine millions, seven hundred and sixty-five thousand, four hundred and thirty-two.
9. Three hundred and forty millions, forty-three thousand, five hundred and sixty-seven.
10. Three hundred and seventy-four billions, four hundred and thirty-eight millions, eight hundred and sixty-two thousand, eight hundred and forty-seven.

FUNDAMENTAL RULES OF ARITHMETIC.

ART. **28.** *Notation* and *Numeration* are the Primary principles of the *four Fundamental* Rules of arithmetic; namely, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION.

These are called *Fundamental Rules*, because all other arithmetical operations are dependent on them.

A *Rule*, in Arithmetic, is a prescribed method of performing an Arithmetical operation.

CHAPTER II.

ADDITION.—SUBTRACTION.—MULTIPLICATION.—DIVISION.

ADDITION.

ART. **29.** ADDITION is the process of finding the *sum* of two or more numbers.

The sign of addition is a short horizontal line bisected by a perpendicular line of the same length; as, $+$. This symbol is called *plus*, and when placed between two quantities, it denotes that they are to be added. Thus, $4+2$ show that four and two are to be added; and is read, *four plus two*.

Two parallel horizontal lines, as $=$, means *equal to* or *equals*, and when placed between two quantites, denotes that they are equal to each other. Thus, $4+2=6$, is read, 4 *plus* 2 *equals* 6.

CASE I.

ART. **30.** Addition of abstract numbers when the sum of each column does not exceed nine.

REMARK.—*Similar concrete* numbers are added the same as though they were abstract numbers, the amount being a concrete number of the same kind. *Dissimilar* concrete numbers cannot be added.

1. What is the sum of 223, 451 and 114?

OPERATION.

Hundreds.	Tens.	Units.	
2	2	3	
4	5	1	
1	1	4	
7	8	8	Sum or Amount.

EXPLANATION.—Write the numbers to be added, so that all the figures of the same denomination shall stand in the same column; and draw a line underneath. Then commencing at the column of units, add each column separately, and place the result directly under it. Thus, 4 and 1 are 5, and 3 are 8 *units*, which place under the column of units. Add the column of *tens*, and of *hundreds* in a similar way, and we obtain for the amount 788.

PROOF.—Begin at the top and add each column downward, the same as you added them upward, if the sums agree the work is right.

2.	3.	4.	5.	6.	7.
134	131	171	315	413	112
512	413	403	481	142	221
342	245	315	202	234	343

8.	9.	10.
1142	4131	10465
3314	1512	43110
2001	3024	22322
1330	201	13101

11. What is the sum of 3141,1202 and 2332?
12. What is the sum of 1674,3102,4011 and 112?
13. What is the sum of 12132,41311,23323 and 1101?
14. What is the sum of 3421,30124,313221 and 1232?
15. What is the sum of 1210+32124+613253+2110301+2101?
16. What is the sum of 1012+32421+613352+2110103+2011?
17. What is the sum of 3121+21+1603+1032?
18. What is the sum of 413+32132+32+4220?
19. What is the sum of 12+3430+40+64213+104?
20. What is the sum of 12+321+4231+821423+12?

PRACTICAL QUESTIONS.

1. If a yoke of oxen is worth 125 dollars, and a cow 32 dollars, what is the value of both?

2. A man bought a load of hay for 5 dollars, a load of wheat for 41 dollars, and some rye for 323 dollars; what was the whole cost?

3. A farmer bought a span of horses for 212 dollars, a yoke of oxen for 132 dollars, and farming implements to the amount of 545 dollars; what was the whole cost?

4. A merchant sold 2131 barrels of flour one month, 11023 barrels the next month, and 6022 barrels the following month; how many barrels did he sell during the three months?

5. A man bought some butter for 123 dollars, some molasses for 310 dollars, some sugar for 1101 dollars, and some flour for 1121 dollars; what was the whole cost?

6. James has 2312 acres of land, John has 21321, and Joseph has 32154; how many acres have they all?

7. A farmer, being asked how many sheep he had, replied, in one field I have 225, in another 2112, in another 1220, and in another 10120; how many had he in all?

8. A farmer raises the following qunatities of grain on four fields, namely: on the first 2115 bushels of wheat, on the second 1110 bushels of rye, on the third 625 bushels of oats, and on the fourth 123 bushels of buckwheat; how many bushels of grain did he raise?

9. A lends to B 1313 dollars; to C 23121 dollars, and has 55125 dollars remaining;—how much money had A at first?

10. A man bought a farm for 4120 dollars, paid 1400 dollars for having it improved, and sold it so as to gain 1150 dollars; for how much did he sell it?

11. A man traveled 214 miles one day, 232 miles the next day, and 320 the third day; how far did he travel in the three days?

12. Mr. Smith owned five farms; the first was worth 23520 dollars, the second 11120 dollars, the third 3200 dollars, and the fourth 32100 dollars; what is the value of the four farms?

13. A drover bought cattle to the amount of 3100 dollars, sheep to the amount of 642 dollars, and a fine horse for 255 dollars; how much did they all cost?

14. A merchant bought groceries to the amount of 3210 dollars, dry goods to the amount of 12210 dollars, and had 32216 dollars remaining; how much had he at first?

15. A had 2310 dollars, B 13250 dollars, C 32118 dollars, and D 321 dollars; how much did they together have?

16. A merchant, on settling up his business, found he owed one man 12326 dollars, another 412 dollars, another 3141 dollars, another 821010 dollars; what was the amount of his debts?

17. A has 2113413 dollars ; B, 534231 dollars ; C, 343 dollars ; and D, 85241002 dollars ;—how many dollars have they together ?

18. In one book there are 1210 pages, in another 235, and in another 1140; how many pages did the three books contain ?

19. A merchant bought books to the amount of 1111 dollars, paper to the amount of 2231 dollars, and dry goods to the amount of 23225 dollars; how much did the whole cost ?

20. A man has a farm worth 1522 dollars, a mortgage worth 23134 dollars, and 5222 dollars of bank sock; how much is he worth ?

CASE II.

ART. **31.** Addition of abstract numbers in general.

1. What is the sum of 4317+346+59+6831+2194+3285 ?

OPERATION.

Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	
	4	3	2	7	
		3	4	6	
			5	9	
	6	8	3	1	
	2	1	9	4	
	3	2	8	5	
1	7	0	4	2	Amount.

EXPLANATION.—Write the numbers to be added as directed in Art. 30.

Begin at units' column and add thus : 5 and 4 are 9, and 1 is 10, and 9 are 19, and 6 are 25, and 7 are 32 *units*,—equal to 3 TENS and 2 *units* ;—place the 2 *units* under the units' column, and *carry*, or add, the 3 *tens* to the tens' column, thus : 3 and 8 are 11, and 9 are 20, and 3 are 23, and 5 are 28, and 4 are 32, and 2 are 34 *tens*,—equal to 3 HUNDREDS and 4 *tens* ;—place the 4 *tens* under the tens' column and add the 3 *hundreds* to the hundreds' column, thus : 3 and 2 are 5, and 1 is 6, and 8 are 14, and 3 are 17, and 3 are 20, *hundreds*,—equal to 2 THOUSANDS and 0 hundreds ;—place the 0 *hundreds* under the hundreds' column and add the 2 *thousands* to the column of thousands, thus : 2 and 3 are 5, and 2 are 7, and 6 are 13, and 4 are 17 *thousands*,—equal to 1 TEN THOUSAND and 7 *thousands*, —place the 7 thousands under the column of thousands, and

the 1 ten thousand on the left of it; and we have for the amount 17042.

PROOF *by the excess of 9's.*—Find the excess of 9's in the sum of the digits of each of the numbers added, and if the excess of 9's in these excesses, equals the excess of 9's in the product, the work may be considered right.

Take for illustration the preceding example.

OPERATION.

4327	=	7	excess.
346	=	4	"
59	=	5	"
6831	=	0	"
2194	=	7	"
3285	=	0	"

Amount, 17042 = 5, is the excess of 9's in the above excesses, also, the excess of 9's in the sum; hence the work is right.

REMARK.—To comprehend this method of proof, as well as that given for the proof of Subtraction, Multiplication and Division, it is necessary to understand the properties of the number 9 explained on page 80th, Art. 74.

2.	3.	4.	5.	6.
3412	7310	782	3241	18243
3410	416	4164	476	32341
218	32	3123	84324	7147
436	4	7182	18472	165
1412	74	119	31421	2342

7. What is the sum of 4862+834+46734+82796+9832 8763?

8. What is the sum of 144+7864+89123+6327+9879?

9. What is the sum of 78639+847796+864321+1487+987?

10. What is the sum of 186+9872+4638+54732+8634+9763+478?

11. Find the sum of 986+834+73257+76324+8763 +9876+4683+9824.

12. Find the sum of 8632+84729+91+7864+9981+ 7632+876324.

13. Find the sum of 71468+3721+863+4902+876+ 8796+87641+763.

14. What is the sum of 8463+721+84632+8468+ 7416+8976+868471+9876741 ?

15. What is the sum of 846832+987649+768321+ 684+9763+84762+9824 ?

16. What is the sum of 18467+982+6849+731247+ 6824721+768417+9763+4214 ?

17. What is the sum of 16824+4768+4734+8686+ 9432+987+98624 ?

18. What is the sum of 23476+1862541+97632+8763 +9768+92+8416+7231 ?

19. What is the sum of 86432+68324+987324+83241 +964 ?

20. What is the sum of 16847+9683+8324+8632+ 749178+76832+1984683+86432147 ?

PRACTICAL QUESTIONS.

1. A father gave to his eldest son 1473 dollars, to his youngest son 3249 dollars, to his oldest daughter 1298 dollars, to his youngest daughter 3998 dollars, and had remaining 7968 dollars ;—how much money had he at first ?

2. Several persons contributed towards building a church. A gave 184 dollars, B gave 273 dollars, C gave 843 dollars, D gave 195 dollars, and E gave 395 dollars ;—how much did they together contribute ?

3. Five brothers had the following sums of money ; A 9789 dollars, B 15450 dollars, C 899 dollars, D 3499 dollars, and E 9999 dollars ;—how much did they together have ?

4. A drover bought 497 sheep one week, 847 the next week, 943 the third week, 1496 the fourth week, and 18550 the fifth week ;—how many sheep did he buy in all ?

5. A gentleman owns a farm worth 3450 dollars, a building lot worth 3759 dollars, a store and lot worth 5868 dollars, a fine horse and carriage worth 775 dollars; what is the amount of his property?

6. From New York to Kingston is 90 miles, from Kingston to Albany is 60 miles, from Albany to Rochester is 251 miles, from Rochester to Buffalo is 75 miles, and from Buffalo to Niagara Falls is 21 miles; how far is it from New York to Niagara Falls?

7. An individual owns a farm worth 2463 dollars, a wood-lot worth 1342 dollars, a store and lot worth 2465 dollars; what is the amount of his property?

8. A gentleman willed his estate to his wife, three sons, and four daughters; to his daughters he willed 3496 dollars a piece; to his sons, each 5785 a piece; and to his wife 4698 dollars;—how much was his estate?

9. The distance on the New York and Erie railroad from New York to Goshen is 59 miles; from Goshen to Narrowsburgh is 63 miles; from Narrowsburgh to Owego is 114 miles; from Owego to Friendship is 137 miles; and from Friendship to Dunkirk is 87 miles. How many miles from New York to Dunkirk?

10. A boy gave for a slate 22 cents; for an arithmetic 50 cents; for an algebra 75 cents; for a grammar 56 cents; and for a geography 125 cents. How much did he give for them all?

11. A butcher sold to one man 436 pounds of meat; to another 3695 pounds; to another 9899 pounds; to another 12485 pounds; and to another 879 pounds. How many pounds did he sell in all?

12. A, B, C, D and E enter into partnership; A puts in 475 dollars; B 846 dollars; C 1495 dollars; D 985 dollars; and E 7864 dollars. How much stock have they in trade?

13. Four persons deposit money in a bank; the first deposits 4490 dollars; the second 5685 dollars; the third 9947 dollars; and the fourth 12470 dollars. How many dollars did they all deposit?

14. Bought of A 346 cords of wood; of B 846 cords;

of C 395 cords ; of D 836 cords ; of E as much as of A and C both ; and of F as much as of B and E both. How many cords of wood did I buy in all ?

15. A produce-dealer has in store at one place 746 bushels of corn, 876 bushels of oats, 395 of rye, and 1247 bushels of potatoes ; at another place 1846 bushels of corn, 3246 bushels of oats, 846 bushels of rye, and 437 bushels of potatoes ; and at another place 199 bushels of corn, 847 bushels of oats, and 849 bushels of potatoes ;—how much produce has he in store ?

16. Macedon was founded 794 years B. C. by Caranus ; Sparta was founded 606 years before Macedon, by Selex ; Corinth, 4 years before Sparta, by Lysippus ; Thebes, 89 years before Corinth, by Cadmus. In what year was Sparta, Corinth, and Thebes founded respectively ?

17. The population of the United States in 1790 was 3729326 ; in 1800 it was 1580427 more ; 1810 it had increased 1930150 more ; in 1820, 2398377 more ; in 1830, 3218241 more ; and in 1840, 4244165 more. What was the population in each of the above mentioned years ?

18. Mr. Harvey, the discoverer of the circulation of the blood, was born in 1578, at Folkstone, in Kent ; George Edwards, the ornithologist, was born 116 years later ; William Herschel, the astronomer, was born 44 years after Edwards ; Henry Clay, the American statesman, was born 39 years after Herschel ;—in what year was each of the above named individuals born ?

19. At the battle of Moskowa there were 13000 Russians killed, 5000 taken prisoners, about 27000 wounded, and 40 generals either killed, wounded or taken prisoners; 2500 of Napoleon's army were killed, 7500 wounded, and 15 generals either killed or wounded. What was the total loss ?

20. At the battle of Waterloo the French lost 40000 men; the Prussians 38000; the Belgians and Dutch 8000; the Hanoverians 3500; and the English about 12000;—how many men were killed in all ?

SUBTRACTION.

ART. **32.** SUBTRACTION is the method of finding the difference between two numbers.

In subtraction there are three terms, the *Minuend*, *Subtrahend*, and *Remainder*. Any two of these being given, the remaining one can be found.

The number from which the other is to be taken is called the *Minuend;* the number to be subtracted from it, the *Subtrahend;* and the result obtained by the operation, the *Remainder*.

A short horizontal line, thus, —, is called *minus*, and is the sign of subtraction. When it is placed between two numbers, it shows that the number on the right of it is to be taken from the one on the left. Thus, 7, (the minuend) — 5, (the subtrahend) = 2, the remainder.

CASE I.

ART. **33.** *Subtraction* of abstract numbers, when each figure of the subtrahend is less than its corresponding figure in the minuend.

REMARK.—The difference of two *similar* concrete numbers is a concrete number of the same kind, and is found in the same way as though they were abstract numbers. But two *dissimilar* concrete numbers can not be taken, the one from the other.

1. From 946 subtract 524.

OPERATION.

	Hundreds.	Tens	Units.
Minuend,	9	4	6
Subtrahend,	5	2	4
Remainder,	4	2	2

EXPLANATION.—Write the less number under the greater, with units under units &c., and draw a line underneath, then proceed thus: 4 units from 6 units, leave 2 units: write the 2 units in units' place, 2 tens from 4 tens leave 2 tens, which write in tens' place, 5 hundreds from 9 hundreds leave 4 hundreds: write the 4 hundreds in the place of hundreds, and we have for the remainder 422.

PROOF.—Add the *remainder* and *subtrahend* together; if their sum is equal to the *minuend*, the work is right.

	2.		3.		4.
From	465	From	842	From	762
Subtract	243	Subtract	511	Subtract	451

	5.		6.		7.
From	549	From	465	From	947
Subtract	334	Subtract	143	Subtract	837

8. From 4631 take 2310.
9. From 16820 take 3410.
10. From 9642 take 8431.
11. From 32478 take 12374.
12. From 96472 take 32361.
13. Subtract 4247 from 7449.
14. Subtract 147302 from 688925.
15. Subtract 234610 from 479824.
16. From 9867412 subtract 4243101.
17. From 1649324 subtract 443121.
18. From 256342 subtract 143242.
19. From 9864324 subtract 8432124.
20. From 9680434 subtract 4530414.

PRACTICAL QUESTIONS.

1. A boy had 36 marbles and gave 24 of them to his playmate; how many had he remaining?

2. Joseph caught 295 quails, and John caught 84; how many more did Joseph catch than John?

3. Jackson had 95 cents and Jane had 73; how many more had Jackson than Jane?

4. Elisha having 447 bushels of potatoes, sold 234 bushels of them to Perry; how many bushels had he remaining?

5. A farmer bought a span of horses for 346 dollars, a yoke of oxen for 135 dollars; how much more did he give for the horses than for the oxen?

6. A drover, having 1465 sheep, sold 1235 of them; how many had he remaining?

7. A gentleman owns a store worth 4695 dollars, and a grist-mill worth 2135 dollars; how much more is the store worth than the grist-mill?

8. A gentleman gave for a house and lot 9899 dollars, for a cotton factory 8495 dollars; how much more did he give for the one than the other?

9. A speculator bought some land for 12897 dollars, a tannery for 10444 dollars; how much more did the land cost than the tannery?

10. A merchant, having 9847 yards of cloth, sold 5844 yards of it; how many yards had he remaining?

11. A drover bought cattle to the amount of 9647 dollars, and sheep to the amount of 5434 dollars; how much more did he give for the cattle than for the sheep?

12. A merchant sold a quantity of goods for 8697 dollars, and by so doing gained 1495 dollars; how much did the goods cost him?

13. A gentleman sold an estate for 7499 dollars, and by so doing gained 1084 dollars; how much did the estate cost him?

14. A farm was sold for 3495 dollars, which was 1032 dollars more than it was worth; how much was it worth?

15. A farmer had 4295 sheep, and 2145 lambs; how many more sheep had he than lambs?

16. A farmer, having 1346 bushels of wheat, sold 1042 bushels of it; how many bushels had he remaining?

17. A merchant, during one year, sold 1247 barrels of molasses, and 2489 barrels of sugar; how many more barrels of sugar did he sell than molasses?

18. A gentleman willed to his son 49865 dollars, and to his daughter 34534 dollars; how much more did he will to his son than to his daughter?

19. A man, driving 1565 sheep to market, on his way sold 435 of them; how many had he remaining?

20. A ship is valued at 69847 dollars, and its cargo at 45837 dollars; how much more is the ship valued at than the cargo?

CASE II.

ART. 34. Subtraction of Abstract numbers in general.
1. From 728 subtract 364.

OPERATION.

	Hundreds. Tens. Units.		Hundreds. Tens. Units.
Minuend,	7 2 8	=	6 (12) 8
Subtrahend,	3 6 4		
Remainder,	3 6 4		

EXPLANATION.—The numbers being properly written down, we proceed thus: 4 units taken from 8 units leave 4 units, which write in unit's place. I cannot take 6 tens from 2 tens; therefore, from the 7 hundreds I take 1 hundred = 10 TENS, and add it to the 2 tens, making 12 *tens*;—6 tens from 12 tens leave 6 tens, which I write in tens' place. I have taken 1 hundred from the seven hundreds, which leaves 6 hundreds; 3 hundreds from 6 hundreds leave 3 hundreds. But for convenience, it is customary to add the 1 hundred to the 3 hundreds, (the next figure in the subtrahend,) and take the sum from the figure in the minuend under which it is placed, which is the same in effect as the above.

NOTE.—The minuend 7 hundreds, 2 tens and 8 units, is = 6 *hundreds*, 12 *tens* and 8 *units*, which form it, absolutely assumes in the mind while giving the above explanation; still it is not necessary to be written except to render the explanation more plain.

PROOF *by the excess of* 9's.—Find the excess of 9's in the sum of the digits of the *remainder*,—also of the *subtrahend*. Then find the excess of 9's in the excesses just found,—if this excess equals the excess of 9's in the *minuend*, the work is right.

Take for illustration the above example:

OPERATION.

728 = 8 excess.
364 = 4 "
———
364 = 4 "
———
8 excess in the subtrahend and remainder, which is the same as the excess in the minuend, therefore the work is right.

	2.		3		4.		5.
From	4642	From	647	From	4621	From	468
Take	2370	Take	352	Take	2432	Take	379

	6.		7.		8.
From	68492	From	7246	From	68243
Subtract	37508	Subtract	5839	Subtract	27359

9. From 8697 subtract 5988.
10. From 1682402 subtract 740482.
11. From 187642 subtract 94837.
12. From 9046 subtract 8074.
13. From 86432 subtract 67821.
14. Subtract 4962 from 7832.
15. Subtract 14829 from 84643.
16. From 4001 subtract 1344.

	OPERATION.		*2nd.*		*3rd.*		*4th.*
	Thousands. Hundreds. Tens. Units.		Thousands. Hundreds. Tens. Units.		Thousands. Hundreds. Tens. Units.		Thousands. Hundreds. Tens. Units.
Minuend,	4 0 0 1	=	3 (10) 0 1	=	3 9 (10) 1	=	3 9 9 (11)
Subtrahend,	1 3 4 4						1 3 4 4
Remainder,	2 6 5 7						2 6 5 7

EXPLANATION.—The numbers being properly arranged, com mence at the right and proceed thus: we cannot take 4 units from 1 unit; therefore I seek 1 from the tens' place, but finding no tens there, I proceed to the hundreds' place, and finding no hundreds there, I take 1 thousand from the 4 thousands and set it in the next place towards the right, which causes the minuend to take the 2nd form. Then take 1 hundred from the 10 hundreds, and set it in the next place towards the right, causes the minuend to take the 3rd form. Now taking 1 ten from the 10 tens, and adding it to the 1 unit causes the minuend to assume the 4th form,—from which we are now prepared to

take the subtrahend. 4 units from 11 units leave 7 units; 4 tens from 9 tens leave 5 tens; 3 hundreds from 9 hundreds leave 6 hundreds; and 1 thousand from 3 thousands leave 2 thousands. Hence the difference of these two numbers is 2657.

REMARK.—The 2nd, 3rd and 4th forms of the minuend serve merely to explain the method of subtracting more clearly, and should, therefore, in practice, be performed in the mind and not be written down. The same result will be obtained by simply adding 10 to the upper figure when it is smaller than the one below it, and carrying, or adding, 1 to the next figure of the subtrahend.

17 From 41007 subtract 34138 ?
18. From 90006 subtract 9994 ?
19. How many are 10000—9 ?
20. How many are 100000—1 ?
21. How many are 89467—84732 ?
22. How many are 760743—249078 ?
23. How many are 4078603—1437908 ?
24. How many are 90807060—60708091 ?
25. How many are 97876757—79787675 ?
26. How many are 20304050—1020304 ?
27. How many are 90857565—20382468 ?
28. How many are 900000—1 ?
29. How many are 909090—1 ?
30. How many are 9080706050—16070809 ?

PRACTICAL QUESTIONS.

1. A gentleman willed to his son 3862 dollars, and to his daughter 5324 dollars; how much more did he will to his daughter than to his son ?

2. In a certain orchard there are 425 apple-trees and 297 plum-trees; how many more apple-trees than plum-trees ?

3. A man traveled 14637 miles during one year, and 9843 miles the next year; how much farther did he travel the first year than the second ?

4. A merchant had 25694 pounds of pork, and sold 19832 pounds of it; how many pounds remained unsold ?

5. A speculator bought a quantity of cotton for 294682 dollars, and sold it for 516390 dollars; how much did he gain ?

6. Gunpowder was invented by Schwartz, in the year 1330; how long was it before the birth of Bonaparte, 1769?

7. George Washington died in the year 1799, at the age of 67; in what year was he born?

8. The mariner's compass was invented at Naples in the year 1302; how long before the discovery of America 1492?

9. Joseph Addison, the poet, was born 1672, and died, 1719; how old was he when he died?

10. Sir William Blackstone, the lawyer, was born 1723, and died, 1780; at what age did he die?

11. Francis Bacon, a universal genius, died in the year 1626, at the age of 65; in what year was he born?

12. Robert Burns, the poet, was born 1759, and died 1796; Lord Byron, the poet, was born 1788, and died 1824. What was the age of each, and how long after the birth of Burns was Byron born?

13. George Edwards, the ornithologist, was born 1694; how long was this before the birth of Harvey, the discoverer of the circulation of the blood, who was born 1578?

14. Massachusetts was settled in 1620, at Plymouth; how many years before the declaration of our National Independence 1776?

15. The Independence of the United States was acknowledged in Europe in 1783; how long was that after the battle of Bunker's Hill, 1775?

16. The first newspaper published in America, at Boston, was in 1704, which was 183 years after Mexico was conquered by the Spaniards; in what year was Mexico conquered?

17. Michael Angelo, an Italian painter, died 1563, at the age of 89; in what year was he born?

18. Benjamin Franklin, the philosopher and statesman, died 1790, at the age of 84; in what year was he born?

19. Galileo, an Italian astronomer, died 1642, at the age of 78; in what year was he born?

20. Luther, the reformer, died 1546, at the age of 63; in what year was he born?

21. Raphael, the prince of painters, an Italian, was born 1483, and died in 1520, which was 6 years after the birth of Titian, another renowned Italian painter; to what age did Raphael live, and in what year was Titian born?

22. Cotopaxi, the highest volcano in the world, is 19408 feet high; how much higher is Sorato, the highest land in America, which is 25380 feet high, than Cotopaxi?

23. Benjamin West, the American painter, was born 1738; how long was this before the death of Robert Fulton, who died in the year 1815?

24. Mount Ararat, (on which Noah's ark rested,) is 12700 feet high; now how much higher is that than mount Washington in New Hampshire, which is 6234 feet in height?

25. St. Peter's Church at Rome, is 450 feet high; how much higher is that than Trinity Church, New York, which is 283 feet in height?

26. Joseph Bonaparte died 1844, at the age of 76; in what year was he born?

27. Dr. Franklin was born in the year 1706, and died in 1790; how old was he when he died?

28. A man, owning 45761 acres of land, sold 23927 acres of it; how many acres had he remaining?

29. A merchant, having 98072 barrels of flour, sold 49267 of them; how many had he remaining?

30. In a certain town there were 24967 inhabitants, which was 5084 more than there were the preceding year; how many were there the preceding year?

31. A merchant sold a quantity of goods for 38967 dollars, which was 873 dollars more than they cost him; how much did they cost him?

32. A man, having 21695 feet of lumber, sold 7962 feet of it; how many feet had he remaining?

33. If I borrow of my neighbor 9673 dollars, and pay him 999 dollars of it; how much remains unpaid?

34. A gentleman sold a farm for 54623 dollars, which was 9240 dollars more than he gave for it; how much did he pay for the farm?

35. A farmer raised 2147 bushels of rye, and 2146

bushels of corn; he sold 943 bushels of the rye, and 789 bushels of the corn;—how much of it remains unsold?

36. A and B bought a farm for 7840 dollars; A paid 2999 dollars, and B the remainder;—how many dollars did B pay?

37. A and B traded farms; A's farm is valued at 9863 dollars, and B's at 7807 dollars;—how much in equity ought B to pay A?

38. Said A to B, I have 4605 sheep; B replied, that he had as many, lacking 298;—how many had B?

39. How many years from 1496, the year in which Algebra was first known in Europe; to 1808, the year in which the first steamboat was put in successful operation by Robert Fulton?

40. A grocer having 346823 dollars' worth of goods, shipped 196832 dollars' worth of them; how many dollars' worth had he remaining?

41. A speculator sold a factory for 35896 dollars, which was 1491 dollars more than it cost him; how much did it cost him?

PRACTICAL QUESTIONS COMBINING ADDITION AND SUBTRACTION.

1. A farmer, having 4632 sheep, sold to A 785, and to B 896; how many had he remaining?

2. A farmer's yearly income was 1679 dollars; he paid for repairing his house 487 dollars; for farming utensils 98 dollars; and for hired help 299 dollars;—how much has he remaining?

3. A man bought a span of horses and a wagon for 987 dollars; he then sold the wagon for 185 dollars, and the horses for 736 dollars;—how much did he lose by the operation?

4. A gentleman, having 697 dollars, deposited 372 dollars in the bank, and spent 197 dollars of it; how much had he remaining?

5. A speculator, having 346821 acres of land, sold to A 637 acres; to B 495; to C 1865; to D 26942; and to E 879 acres;—how many acres had he left?

6. There is a farm consisting of 946 acres; 35 acres of which is planted with corn and potatoes; 140 acres sown with rye; 180 acres with oats; 98 with wheat; 212 is pastured, and the remainder is meadow. How many acres of meadow?

7. A lady, having 467 dollars, paid for a bonnet 24 dollars; for a shawl 85 dollars; for a silk dress 90 dollars; and for some delaines 112 dollars;—how much had she remaining?

8. A market-woman, having 234 oranges, sold to one person 12 of them; to another 46; to another 54; to another 32; and to another 15;—how many had she remaining?

9. A farmer, having 897 sheep, sold to A 150 of them; to B 160; to C 284; and to D 294;—how many had he remaining?

10. A drover, having 197 cattle, sold 112 of them, and bought 87 more; how many had he then?

11. In a certain army there are 4560 men: in a battle 646 of them were killed, 497 of them wounded, and 148 of them deserted; how many were left?

12. A farmer, having 847 bushels of grain, sold to A 132 bushels; to B 112; to C 184; and gave to the poor 212 bushels;—how many bushels had he remaining?

13. An individual traveled by railroad 497 miles, and designed to return on foot; the first day he traveled 69 miles; the second 84; the third 59; the fourth 47 miles; the fifth day he took the cars and arrived home. How far did he go the last day?

14. A man willed an estate of 560048 dollars to his two children and wife, as follows: to his son 230645 dollars; to his daughter 88999 dollars; and to his wife the remainder. How much did he will to his wife?

15. A man laid out 98000 dollars in speculation; the first year he gained 1847 dollars; the second year 1987 dollars; the third year he lost 8044 dollars. How much did he lose by the operation?

16. A merchant, having 89776 barrels of flour, sold to A 967 barrels; to B 1743 barrels; to C 6842 barrels;

to D 14625 barrels; and to E the remainder. How many barrels did E receive?

17. Four persons A, B, C and D propose to purchase a manufactory, valued at 97802 dollars. A is to pay 4990 dollars, B 12647 dollars, C 19682 dollars, and D the remainder; what sum will D have to pay?

18. Having in my possession 8960 dollars, I wish to know how much I must add to this sum, to be able to purchase a farm worth 18970 dollars, and save 497 dollars for other purposes?

19. A had 448 oxen; B had 212 more than A; and C had as many as A and B together, lacking 184;—how many had B and C respectively?

20. A has 470 dollars more than B, and 245 dollars less than C, who has 2490 dollars; and D has as much as A and B together. How many dollars have A, B and D respectively?

21. John has 240 sheep more than Joseph, and 125 less than James, who has 485; and Jackson has as many as John and Joseph together, lacking 320 sheep. How many sheep have John, Joseph and Jackson respectively?

MULTIPLICATION.

ART. **35.** MULTIPLICATION is a concise method of computing the amount of any number taken as many times as there are units in another number.

There are three terms employed in multiplication; the *Multiplicand*, the *Multiplier*, and the *Product;* any two of which being given the remaining one can be found.

ART. **36.** The *Multplicand* is the number taken. The *Multiplier* is the number that shows how many times the multiplicand is taken. The *Product* is the answer or result obtained. The multiplicand and multiplier are also called *Factors* of the product.

ART. **37.** The *multiplier* can never be a CONCRETE num-

ber, as it merely expresses the number of times the multiplicand is taken. The *product* will be of the same denomination as the *multiplicand.*

ART. **38.** The sign of multiplication is two short lines of equal length bisecting each other at an angle of 45 degrees with the horizon; thus, ×, and is sometimes called INTO. This sign being placed between two numbers shows that they are to be multiplied, the one by the other. Thus, 6×8=48, indicates that 6 is to be multplied by 8, or 8 to be multiplied by 6, (as the case may require,) and that the product equals 48.

MULTIPLICATION TABLE.

2× 0= 0	3× 0= 0	4× 0= 0	5× 0= 0	6× 0= 0	7× 0= 0
2× 1= 2	3× 1= 3	4× 1= 4	5× 1= 5	6× 1= 6	7× 1= 7
2× 2= 4	3× 2= 6	4× 2= 8	5× 2=10	6× 2=12	7× 2=14
2× 3= 6	3× 3= 9	4× 3=12	5× 3=15	6× 3=18	7× 3=21
2× 4= 8	3× 4=12	4× 4=16	5× 4=20	6× 4=24	7× 4=28
2× 5=10	3× 5=15	4× 5=20	5× 5=25	6× 5=30	7× 5=35
2× 6=12	3× 6=18	4× 6=24	5× 6=30	6× 6=36	7× 6=42
2× 7=14	3× 7=21	4× 7=28	5× 7=35	6× 7=42	7× 7=49
2× 8=16	3× 8=24	4× 8=32	5× 8=40	6× 8=48	7× 8=56
2× 9=18	3× 9=27	4× 9=36	5× 9=45	6× 9=54	7× 9=63
2×10=20	3×10=30	4×10=40	5×10=50	6×10=60	7×10=70
2×11=22	3×11=33	4×11=44	5×11=55	6×11=66	7×11=77
2×12=24	3×12=36	4×12=48	5×12=60	6×12=72	7×12=84

8× 0= 0	9× 0= 0	10× 0= 0	11× 0= 0	12× 0= 0
8× 1= 8	9× 1= 9	10× 1= 10	11× 1= 11	12× 1= 12
8× 2=16	9× 2= 18	10× 2= 20	11× 2= 22	12× 2= 24
8× 3=24	9× 3= 27	10× 3= 30	11× 3= 33	12× 3= 36
8× 4=32	9× 4= 36	10× 4= 40	11× 4= 44	12× 4= 48
8× 5=40	9× 5= 45	10× 5= 50	11× 5= 55	12× 5= 60
8× 6=48	9× 6= 54	10× 6= 60	11× 6= 66	12× 6= 72
8× 7=56	9× 7= 63	10× 7= 70	11× 7= 77	12× 7= 84
8× 8=64	9× 8= 72	10× 8= 80	11× 8= 88	12× 8= 96
8× 9=72	9× 9= 81	10× 9= 90	11× 9= 99	12× 9=108
8×10=80	9×10= 90	10×10=100	11×10=110	12×10=120
8×11=88	9×11= 99	10×11=110	11×11=121	12×11=132
8×12=96	9×12=108	10×12=120	11×12=132	12×12=144

CASE I.

ART. **39.** *Multiplication* of abstract numbers, when the multiplier does not exceed 9.

1. Multiply 846 by 8.

OPERATION.

	Thousands.	Hundreds.	Tens.	Units.
Multiplicand,		8	4	6
Multiplier,				8
Product,	6	7	6	8

EXPLANATION.—Write the numbers down, placing units under units; then proceed from right to left; thus, 8 times 6 units are 48 units, or 4 *tens* and 8 units;—place the 8 units in units' place, and reserve the 4 *tens* to add to the next product, 8 times 4 tens are 32 tens, and 4 tens added are 36 tens, or 3 *hundreds* and 6 *tens*;—place the 6 tens in tens' place and reserve the 3 hundreds to add to the next product, 8 times 8 hundreds are 64 hundreds, and 3 hundreds added are 67 hundreds, or 6 *thousands* and 7 *hundreds*, which write down; and we have for the product 6768.

PROOF.—Multiply the multiplier by the multiplicand; if the product thus obtained, equals the first product the work is presumed to be right.

2. Multiply 348 by 2.
3. Multiply 483 by 3.
4. Multiply 684 by 4.
5. Multiply 6482 by 4.
6. Multiply 14682 by 5.
7. Multiply 18623 by 6.
8. Multiply 38943 by 7.
9. Multiply 28462 by 8.
10. Multiply 8946 by 7.
11. Multiply 7683 by 6.
12. Multiply 9898 by 9.
13. Multiply 6847 by 3.
14. Multiply 94762 by 6.
15. Multiply 88992 by 7.
16. Multiply 33449 by 8.
17. Multiply 884682 by 9.
18. Multiply 99999 by 5.
19. Multiply 897654 by 7.
20. Multiply 123456789 by 8.

PRACTICAL QUESTIONS.

1. A man sold 195 sheep, at 3 dollars a piece; how much did he receive for them?

2. What cost 184 barrels of flour, at 6 dollars a barrel?

3. What cost 1987 acres of land, at 9 dollars an acre?

4. What cost 4786 barrels of sugar, at 9 dollars a barrel?

5. In 1 mile there are 5280 feet; how many feet in 5 miles?

6. In 1 mile there are 1760 yards; how many yards in 5 miles?

7. If 9 men can mow a certain meadow in 18 days; in how many days can one man do the same?

8. If 6 masons can build a certain wall in 149 days; in how many days can one mason build the same wall?

9. If 460 bushels of oats will feed 1 horse 11 months; how many bushels will be required to feed 8 horses the same time?

10. Bought 245 cords of wood, at 7 dollars a cord. What did the whole cost?

11. A farmer sold 8 horses, at 253 dollars a piece; how many dollars did he receive for them?

12. A lady bought 189 yards of ribbon, at 6 cents a yard; how much did it all cost her?

13. What cost 1786 boxes of raisins, at 3 dollars a box?

14. If a steamship can go 395 miles in 1 day, how far can she go in 9 days?

15. A merchant bought 2864 hats, at 4 dollars a piece; how much did he pay for them all?

16. A farmer sold 9 fat oxen, at 185 dollars a piece; how much did he receive for them all?

17. In 1 day there are 1440 minutes; how many minutes in 8 days?

18. In 1 day there are 86400 seconds; how many seconds in 5 days?

19. If one man receive 4 dollars a week, how much will an army of 35680 men receive in 6 weeks?

20. At 2 dollars a day, each; how much will it cost, to board 685 men 7 days?

CASE II.

ART. **40.** *Multiplication* of abstract numbers in general?

1. Multiply 437 by 56.

OPERATION.

Hundreds.	Tens.	Units.
4	3	7
	5	6

```
 2622
2185
-----
24472
```

EXPLANATION.—Write the numbers down so that units stand under units, tens under tens, &c. Begin at the right and proceed, thus,—6 times 7 units are 42 units, or 4 *tens* and 2 *units;* write the 2 units in units' place, and reserve the 4 tens to add to the next product. 6 times 3 tens are 18 tens, and 4 tens added are 22 tens, or 2 *hundreds* and 2 *tens*, &c. We next multiply by the 5 *tens*. For convenience we say 5 times 7 are 35, and place the 5 under the multiplier, 5, that is in tens' place, and reserve the three hundreds to add to the next product, &c. But instead of 5 times 7, &c., it is, 50 times 7 *units* = 350 units, or 3 *hundreds*, 5 *tens* and 0 units. I therefore placed the 5 *tens* in tens' place, where you perceive it belongs. We proceed in the same way to explain why we place the right hand figure of the product in the third, or hundreds' place when multiplying by that figure, &c.

PROOF *by the excess of* 9's.—Find the excess of 9's in *each* FACTOR. Then if the excess of 9's in the product of these excesses, equals the excess of 9's in the *product* of the two factors, the work is right.

Take for illustration the preceding example:

OPERATION.

Factors,	437 = 5	excess.
	56 = 2	"
	2622 — 1	excess in the product of the excesses.
	2185	
Product,	24472 = 1	excesses in the product of the factors.

EXPLANATION.—Commence at the left of the first factor, (the multiplicand,) and add, thus, 4+3+7 are 14, or 1 nine and 5

units, the excess of 9's in that factor. Add the second factor, (the multiplier,) 5+6 are 11, or 1 nine and 2 units, the excess of 9's in that factor. The product of these excesses is 10, or 1 nine and 1 unit, the excess of 9's in the product of the excesses. Add the total product, 2+4+4+7+2 are 19, or 2 nines and 1, the excess of 9's in the product of the factors. This excess equals the required excess; hence the work is right.

2. Multiply 4624 by 35.
3. Multiply 3846 by 39.
4. Multiply 8462 by 47.
5. Multiply 7846 by 147.
6. Multiply 3976 by 183.
7. Multiply 2243 by 144.
8. Multiply 2882 by 414.
9. Multiply 1414 by 323.
10. Multiply 2463 by 382.
11. Multiply 8632 by 132.
12. Multiply 4862 by 897.
13. Multiply 9876 by 678.
14. Multiply 4567 by 7654.
15. Multiply 1234 by 4321.
16. Multiply 8362 by 8496.
17. Multiply 146832 by 8376.
18. Multiply 36847 by 8324.
19. Multiply 1384697 by 476324.
20. Multiply 897654321 by 123456789.

PRACTICAL QUESTIONS.

1. If 15 men can build a certain wall in 235 days, how long will it take 1 man to do it?

2. If 45 men can accomplish a certain piece of work in 360 days by working 8 hours a day, how many days will it take one man to do the same by working 4 hours a day?

3. If 360 bushels of oats will last 185 horses 3 days, how long will it last 1 horse?

4. A drover bought 685 oxen at 104 dollars a piece; what was the cost of all of them?

5. A merchant bought 25 pieces of broadcloth, each piece containing 48 yards, at 9 dollars a yard. How much did he pay for the whole?

6. If a steamship can sail 18 miles in 1 hour, how far can she sail in 34 days of 24 hours each?

7. A speculator bought 8968 acres of land, at 195 dollars an acre. How much did the whole cost him?

8. In 1 furlong there are 660 feet; how many feet in 8 furlongs, (1 mile)?

9 How many pounds of flour are there in 395 barrels; there being 196 pounds in each barrel?

10. What is the value of 346 shares of railroad stock, at 125 dollars a share?

11. How many pages are there in 5896 books, there being 394 pages in each book?

12. A speculator bought 302 cattle, and 293 times as many sheep; how many sheep did he buy?

13. If a garrison of men consume 987 pounds of beef in 1 day; how many pounds will a garrison containing twice as many men consume in 365 days?

14. Farmer A has 245 acres, sowed with wheat, which produces 32 bushels to the acre. Farmer B has 360 acres, sowed with wheat, which produces 25 bushels to the acre. What quantity of wheat was raised by A and B respectively?

15. A speculator bought 146 head of oxen; 230 head of cows; and 69 head of calves. He made a profit of 16 dollars a head on the oxen; 12 on the cows; and 5 on the calves. How much did he gain on the oxen, cows, and calves respectively?

16. A merchant bought 12 boxes of linen, each containing 25 pieces, and each piece containing 36 yards, at 65 cents a yard. How many pieces, and how many yards did he buy, and how much did it all cost him?

17. A has 395 acres of land, worth 27 dollars an acre; and B has 493 acres, worth 19 dollars an acre. What is the value of each of their farms?

18. In a certain orchard there are 26 rows of apple-trees and 36 trees in each row. How many apples would there be in the orchard, allowing 2595 apples to each tree?

19. A farmer purchased five tracts of land, each containing 395 acres, at 95 dollars an acre. What was the whole cost?

20. The circumference of the earth is nearly 25000 miles, the distance to the sun is 3800 times as much. What is the distance to the sun?

ART. **41.** A COMPOSITE number is one that can be produced by multiplying two or more numbers together, each of which is greater than a unit.

Thus, 15 is a *composite* number, as it can be produced by multiplying together the numbers 3 and 5. The 3 and 5 are called the *factors* of 15.

ART. **42.** When the multiplier is a composite number, resolve it into two or more factors, then multiply the multiplicand by one of these factors, and the product thus obtained by another factor, and so on, until all the factors have been used as a multiplier. The last product will be the answer sought

1. Multiply 743 by 35.

OPERATION.

$$\begin{array}{r} 743 \\ 7 \\ \hline 5201 \\ 5 \\ \hline 26005 \end{array}$$

EXPLANATION.—The factors of 35 are 5 and 7; hence, multiplying by 5 and 7, or 7 and 5, will produce the same result as multiplying by 35; since $5 \times 7 = 35$.

2. What cost 325 bushels of potatoes, at 63 cents a bushel?

3. What cost 437 melons, at 21 cents a piece?

4. What cost 395 yards of muslin, at 27 cents a yard?

5. What cost 49 sheep, at 425 cents a head?

6. What cost 77 horses, at 245 dollars a piece?

7 What cost 15 acres of land, at 595 dollars an acre?

8. What cost 70 bush. of wheat, at 145 cents a bushel?
9. What cost 18 pounds of opium, at 845 cents a pound?
10. What cost 21 books at 95 cents a piece?

ART. **43.** *Multiplication* of abstract numbers, when there are ciphers on the right of the multiplier or multiplicand, or both.

1. Multiply 3464 by 2430000.

OPERATION.

```
      3464
   3430000
----------
    10392
   13856
  10392
----------
11881520000
```

EXPLANATION.—Write the numbers down, so that the right hand significant figures of the two factors shall come one under the other; then multiply as in Case 2, Article 40, and bring the ciphers down on the right of the product.

REMARK.—This method of operation is a particular case under Art. 41. For in fact the number 3430000, is resolved into the two factors 343 and 10000. We first multiplied the minuend by 343, and then the product thus obtained by 10000, which is done by merely adding four ciphers.

2. Multiply 2460000 by 432000

OPERATION.

```
Multiplicand,      2460000
Multiplier,         432000
                 -------------
                   492
                  738
                 984
                 -------------
Product,         1062720000000
```

3. Multiply 232 by 10.
4. Multiply 682 by 100.
5. Multiply 543 by 1000.
6. Multiply 4321 by 10000.
7. Multiply 3261 by 100000.
8. Multiply 246800 by 100000.
9. Multiply 2326000 by 43000.
10. Multiply 3680200 by 4863000.
11. Multiply 12360000 by 43298000.
12. Multiply 326000200 by 20046000

PRACTICAL QUESTIONS COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. If a wagon cost 48 dollars, a yoke of oxen 3 times as much, lacking 54 dollars, and a span of horses as much as the wagon and oxen together; what was the cost of the oxen and horses respectively, and of all?

2. A man paid for building his house 2460 dollars; for his farm 4 times as much, lacking 986 dollars; and for his furniture, 122 dollars less than he paid for building his house. How much did he pay for all, and for each respectively?

3. Two persons start together from the same place, and travel in the same direction. One proceeds at the rate of 35 miles a day; the other at the rate of 42 miles a day. What distance will they be apart at the end of 45 days?

4. Bought 197 acres of land, at 47 dollars an acre; at another time double the number of acres, at double the price per acre, lacking 12 dollars; and at another time as many acres as I had already bought, at 145 dollars an acre. How many acres did I buy, and how much did it cost me?

5. A farmer purchased three tracts of land: the first contained 195 acres; the second 6 times as much, lacking 203 acres; the third as much as the first and second together, and 45 acres more. How many acres did the farmer purchase, and what did the whole amount to, at 45 dollars an acre?

6. A planter sold 465 bales of cotton, at 35 dollars a bale; and out of the proceeds bought 18 mules, at 65 dollars each; 6 span of horses, at 147 dollars a span; and 4 yoke of oxen, at 95 dollars a pair. How much money had he left from the sale of his cotton?

7. Mr. B.'s yearly income is 2890 dollars: he pays for house-rent 265 dollars; his family expenses amount to 7 times as much, lacking 199 dollars. How much does he save annually?

8. A man, having 6894 dollars, paid out of it 1684 dollars for a farm; twice as much, lacking 1999 dollars,

for building a house; and the remainder, lacking 989 dollars, for farming utensils and furnishing his house. What was the cost of the farm, the house, and of the farming utensils and furniture of the house, respectively?

9. A has 789 sheep; B has 4 times as many, lacking 999; and D has 45 sheep more than A and B together. How many sheep has B and D respectively, and how many have they all?

10. A is worth 8967 dollars; B is worth 285 dollars more than A; and C is worth as much as A and B together, lacking 3794 dollars. How much are B and C worth respectively?

11. In an army of 8645 men, 1864 men were killed in an action; and 4 times as many wounded, lacking the number that deserted, which was 984. How many men were wounded, and how many remained in the army?

12. A certain house is worth 1460 dollars; the farm on which it stands is worth 5 times as much, + 896 dollars; and the stock on the farm is worth 4 times as much as the house, lacking 1980 dollars. What is the value of all, and of the farm and stock respectively?

13. A lends B 12804 dollars. B let A have bank stock to the amount of 2042 dollars; a farm for 5 times as much as the bank stock, lacking 989 dollars; and is to pay the remainder in cash. How much cash ought B to pay A?

14. John has 240 sheep; James 15 times as many + 146; and Joseph 8 times as many as both John and James, lacking 8999. How many sheep has James and John, and how many have they all?

15. If a cow cost 43 dollars; a horse 5 times as much; and a farm 9 times as much as the cow and horse together, lacking 36 dollars; how much more will the farm cost than 5 horses and 9 cows, at the same rate?

16. If a quantity of sugar cost 1465 dollars; a store 15 times as much, lacking 9999 dollars; and the lot on which the store stands 2 times as much as the sugar and store together, + 146 dollars; what will be the cost of all, and of the store and lot respectively?

17. Said Martha to Baldwin, I am worth 245 dollars; Baldwin replies, that is exactly 1 fifth as much as Ann is worth, and 1 twelfth as much as I am worth. How much are Ann and Baldwin together worth?

18. If a quantity of flour cost 2864 dollars; the store in which it is deposited, 14 times as much, lacking 984 dollars; and the lot on which the store stands 3 times as much as the flour and store together, + 183 dollars;—what will be the cost of all, and of the store and lot respectively?

19. A merchant bought 12 pieces of broadcloth, each piece containing 32 yards, at 5 dollars a yard for the two pieces; 6 dollars a yard for six pieces; and 8 dollars a yard for the remaining four pieces. He sold it all, at 7 dollars a yard, did he gain or lose, and how much?

DIVISION.

ART. **44.** DIVISION teaches how to find the number of times, or part of a time that one number is contained in another.

There are three terms employed in division, the *Divisor*, *Dividend* and *Quotient*. That which is left, (if any,) after the division, is called the *Remainder;*—we have not called it a distinct term of division, as it is a part of the dividend.

The *Divisor* is the dividing number. The *Dividend* is the number to be divided. The *Quotient* is the number of times the dividend contains the divisor.

Division is indicated by the symbol, $\div$. This sign when placed between two quantities, shows that the number an the left is to be divided by the one on the right. Thus, $8 \div 4 = 2$; shows that 8 is to be divided by 4, and that the quotient is 2. In the division of *concrete numbers*, the *divisor* is always considered ABSTRACTLY. The quotient is a concrete number of the same kind as the dividend.

Division is also indicated by writing the divisor under the dividend; thus, $\frac{12}{4} = 3$.

DIVISION TABLE.

2÷2= 1	3÷3= 1	4÷4= 1	5÷5= 1	6÷6= 1	7÷7= 1
4÷2= 2	6÷3= 2	8÷4= 2	10÷5= 2	12÷6= 2	14÷7= 2
6÷2= 3	9÷3= 3	12÷4= 3	15÷5= 3	18÷6= 3	21÷7= 3
8÷2= 4	12÷3= 4	16÷4= 4	20÷5= 4	24÷6= 4	28÷7= 4
10÷2= 5	15÷3= 5	20÷4= 5	25÷5= 5	30÷6= 5	35÷7= 5
12÷2= 6	18÷3= 6	24÷4= 6	30÷5= 6	36÷6= 6	42÷7= 6
14÷2= 7	21÷3= 7	28÷4= 7	35÷5= 7	42÷6= 7	49÷7= 7
16÷2= 8	24÷3= 8	32÷4= 8	40÷5= 8	48÷6= 8	56÷7= 8
18÷2= 9	27÷3= 9	36÷4= 9	45÷5= 9	54÷6= 9	63÷7= 9
20÷2=10	30÷3=10	40÷4=10	50÷5=10	60÷6=10	70÷7=10
22÷2=11	33÷3=11	44÷4=11	55÷5=11	66÷6=11	77÷7=11
24÷2=12	36÷3=12	48÷4=12	60÷5=12	72÷6=12	84÷7=12

8÷8= 1	9÷9= 1	10÷10= 1	11÷11= 1	12÷12= 1
16÷8= 2	18÷9= 2	20÷10= 2	22÷11= 2	24÷12= 2
24÷8= 3	27÷9= 3	30÷10= 3	33÷11= 3	36÷12= 3
32÷8= 4	36÷9= 4	40÷10= 4	44÷11= 4	48÷12= 4
40÷8= 5	45÷9= 5	50÷10= 5	55÷11= 5	60÷12= 5
48÷8= 6	54÷9= 6	60÷10= 6	66÷11= 6	72÷12= 6
56÷8= 7	63÷9= 7	70÷10= 7	77÷11= 7	84÷12= 7
64÷8= 8	72÷9= 8	80÷10= 8	88÷11= 8	96÷12= 8
72÷8= 9	81÷9= 9	90÷10= 9	99÷11= 9	108÷12= 9
80÷8=10	90÷9=10	100÷10=10	110÷11=10	120÷12=10
88÷8=11	99÷9=11	110÷10=11	121÷11=11	132÷12=11
96÷8=12	108÷9=12	120÷10=12	132÷11=12	144÷12=12

SHORT DIVISION.

ART. **45.** Division of abstract numbers, when the divisor does not exceed 12.

1. Divide 8245 by 5.

OPERATION.

Thousands. Hundreds. Tens. Units.

Divisor. Dividend.
5) 8 2 3 5

Quotient, 1 6 4 7

EXPLANATION.—Write the divisor at the left of the dividend, with a curved line between them; and under the dividend draw a horizontal line. Begin at the left and proceed; thus, 5 is contained in 8 thousands, 1 thousand times and 3 thousands remaining. Write the 1 thousand down by placing the 1 under the figure divided. The remainder, 3 thousands, added to 2 hundreds, (which is the same as prefixing the 3 to next figure,) are 32 hundreds. 5 is contained in 32 hundreds, 6 hundreds times and 2 hundreds remaining. Write the 6 hundreds down by placing the 6 under the last figure

divided. The remainder, 2 hundreds, added to 4 tens, (which is the same as prefixing the 2 to the next figure,) are 23 tens. 5 is contained in 23 tens, 4 tens times and 4 tens remaining Write the 4 tens under the last figure divided. The remainder, 3 tens added to 5 units, are 35 units. 5 is contained in 35 units, 7 units times, which place under the last figure divided; and we obtain for the quotient, 1647.

REMARK.—After the pupil *thoroughly understands* the above explantion, the following may be adopted.

OPERATION.

Divisor. Dividend.
5) 8235

Quotient, 1647

EXPLANATION.—5 is contained in 8, 1 and 3 remaining. Write down the 1 and prefix the remainder to the next figure. 5 is contained in 32, 6 times and 2 remaining. Write down the 6. 5 is contained in 23 4 times and 3 remaining. 5 is contained in 35, 7 times and no remainder.

2. Divide 1467 by 7.

OPERATION.

Divisor. Dividend
7) 1 4 6 7 7

Quotient, 2 0 9 6—5 remaider.

REMARK.—The remainder 5 may be divided by 7, and written with the quotient; thus, 2096$\frac{5}{7}$, or mentioned simply as a remainder, as occasion requires.

PROOF.—*Multiply the divisor by the quotient and add in the remainder, if there be any.* If this sum is equalto the dividend, the work is right.

REMARK.—From what we have already learned, we discover that division is the reverse of multiplication, and that either may be used to verify, or prove the correctness of the work of the other.

3. Divide 4682 by 2.
4. Divide 3468 by 2.
5. Divide 7639 by 3.
6. Divide 8472 by 4.
7. Divide 89631 by 4.
8. Divide 142632 by 7.
9. Divide 34682 by 6
10. Divide 24673 by 5.
11. Divide 147268 by 5.

12. Divide 476846 by 9.
13. Divide 476342 by 8.
14. Divide 8462324 by 8.
15. Divide 8496723 by 9.
16. Divide 8467232 by 8.
17. Divide 246832 by 10.
18. Divide 467232 by 11.
19. Divide 2468324 by 12.

PRACTICAL QUESTIONS.

1. If 9 acres of land cost 2250 dollars, what will 1 acre cost?

2. If 8 horses cost 1696 dollars, what will 1 horse cost?

3. If a man travel 693 miles in 9 days, how far does he travel in 1 day?

4. Divide 1648 acres of land equally among 8 individuals.

5. If 6 horses sell for 1332 dollars, what will be the average sum received for each?

6. A man bought 12 tons of hay for 192 dollars; how much did he pay a ton?

7. A boy sold 11 rabbits for 286 cents; how much did he receive a piece?

8. A girl spent 342 cents for oranges, at 3 cents a piece; how many oranges did she buy?

9. Divide 68425 dollars equally among 7 sons.

10. How many barrels of flour, at 6 dollars a barrel, can be bought for 25218 dollars?

11. At 8 dollars a cord, how many cords of wood can be bought for 1928 dollars?

12. At 5 dollars a barrel, how many barrels of cider can be bought for 1465 dollars?

13. If in 1 week there are 7 day, how many weeks are there in 365 days, (one year)?

14. A man bought a store for 3792 dollars, which was 3 times as much as his house cost him; how much did his house cost him?

15. A drover bought 12 oxen for 1764 dollars; how much was the average cost of each?

16. A laborer worked 12 months for 288 dollars; how much did he receive a month?

17. A is worth 15795 dollars, which is 5 times as much as B is worth, and B is worth 3 times as much as C; how much are B and C worth respectively?

18. A's house cost 2358 dollars, which is 3 times as much as the furniture of the house cost; what was the cost of the furniture?

19. Says A to B, I have 74 sheep; B replies, that is just 1 tenth of my number, which is 4 times C's number; how many sheep has C?

20. Edward is worth 2000 dollars, which is 3 times Luther's fortune, lacking 700 dollars: and Caleb is worth 4 times as much as Edward and Luther together + 400 dollars. What is the fortune of each?

Long Division.

Art. 46. *Division* of abstract numbers in general.

1. Divide 4379 by 24.

OPERATION.

```
            Thousands.
             Hundreds.
              Tens.
               Units.
Divisor.  Dividend.  Quotient.
  24)      4379      (100
           2400        80
           ----         2
           1979      -----
           1920      182 11/24
           ----
             59
             48
             --
Remainder,   11
```

Explanation.—Write the divisor on the left of the dividend, and the quotient on the right, separating them with a curved line, and proceed thus: 24 is contained in 43 hundreds and 79, 1 hundred times; write the 100 in the quotient, 100 times 24 is 2400, which being subtracted from the dividend, leaves 1979, 24 is contained in 1979, 80 times; write the 80 in the quotient, 80 times 24 are 1920, which being subtracted from the 1979, leaves 59. 24 is contained in 59, 2 times; write the 2 in the quotient. 2 times 24 are 48, which

being subtracted from the 59, leaves 11. Dividing the 11 by 24 we have $\frac{11}{24}$, which annex to the quotient.

REMARK.—After the pupil comprehends the above explanation, the following may be adopted.

OPERATION.

Divisor.	Dividend.	Quotient.
24)	4379	$(182\frac{11}{24}$
	24	
	197	
	192	
	59	
	48	
	11	

EXPLANATION.—24 is contained in 43, 1 time. Write the 1 in the quotient. 1 times 24 is 24, which being subtracted from 43, leaves 19. Bring down the next figure of the dividend. 24 is contained in 197, 8 times; write the 8 in the quotient. 8 times 24 are 192, which being subtracted from 197, leaves 5. Bring down the next figure of the dividend. 24 is contained in 59, 2 times. Write the 2 in the quotient, 2 times 24 are 48, which being subtracted from 59, leaves 11. Divide the remainder by 24; thus, $\frac{11}{24}$; and place it in the quotient.

PROOF *by the excess of 9's.*—Find the excess of 9's in the divisor and quotient respectively, and also, the excess of 9's in the product of these two excesses; and if this last excess is equal to the excess of 9's in the difference between the dividend and remainder, the work is right.

Take for illustration the above example:

Divisor,	24	= 6 excess.	
Quotient,	182	= 2 excess.	
Dividend,	4379	12 = 3	excess.
Remainder,	11		
Difference,	4368 =	3	excess.

2. Divide 4368 by 13.
3. Divide 3690 by 15.
4. Divide 8041 by 17.
5. Divide 5490 by 15.
6. Divide 1242 by 27.
7. Divide 66384 by 24.

8. Divide 108220 by 28.
9. Divide 18336 by 24.
10. Divide 2841 by 29.
11. Divide 3570 by 15.
12. Divide 6048 by 72.
13. Divide 3607344 by 24.
14. Divide 949073 by 73.
15. Divide 9334949 by 307.
16. Divide 789591 by 213.
17. Divide 86431 by 342.
18. Divide 986321 by 412.
19. Divide 2364 by 82.
20. Divide 146832 by 147.
21. Divide 246832 by 432.
22. Divide 846324 by 1432.
23. Divide 98476324 by 1463.
24. Divide 1476324 by 1482.
25. Divide 47632463 by 24801.
26. Divide 476784631 by 1472
27. Divide 48468234 by 423.
28. Divide 123456789 by 846.
29. Divide 987654321 by 146.
30. Divide 987644698321 by 3223.

PRACTICAL QUESTIONS.

1. If one man can accomplish a certain piece of work in 494 days, how many days will it take 38 men to do the same?

2. If 99 sheep cost 396 dollars, what will 1 sheep cost?

3. If 97 acres of land cost 22989 dollars, how much is that an acre?

4. What cost 1 barrel of flour, if 36 barrels cost 288 dollars?

5. If in 89 books there are 28035 pages, how many pages on an average in a book?

6. If an iceberg move at the rate of 25 miles a day, how many days would it be in moving from the north pole to the equator, it being about 6250 miles?

7. If a horse can travel 54 miles in a day, how many days will it take it to travel 10854 miles?

8. If 15 months' wages amount to 525 dollars, how much is that a month?

9. If 347 quails are sold for 7287 cents, how much is that a piece?

10. If 38 baskets of peaches are sold for 2850 cents, how much is that a basket?

11. A drover bought cattle, at 37 dollars a-head, and paid for them 8732 dollars, how many did he buy?

12. How many barrels of molasses, at 17 dollars a barrel, can be bought for 3604 dollars?

13. How many pieces of cloth, at 95 dollars a piece, can be bought for 3385 dollars?

14. If 63 gallons make 1 hogshead, how many hogsheads will 1449 gallons make?

15. For 1016 dollars, how many yards of broadcloth can be bought, at 8 dollars a yard?

16. If a steamship can cross the Atlantic Ocean, a distance of 3000 miles, in 9 days; how many miles does the ship go daily?

17. Baldwin's income is 2555 dollars a year, how much is that a day, allowing the year to consist of 365 days?

18. Walter purchased a farm containing 235 acres, for 4230 dollars; how many dollars did he pay an acre?

19. In how many days could 27 men accomplish the same amount of work, that 1 man could in 594 days?

20. If a railroad car move at the rate of 625 miles a day, in how many days would it go around the earth, the distance being about 25000 miles?

ART. **47.** To divide one number by another, when the divisor is a composite number.

Resolve the number into two or more factors, then divide by one of these factors, and the quotient thus obtained by another factor,—proceed in the same way till all the factors have become divisors, and the last quotient obtained will be the answer required.

ART. 48. To find the true remainder.

To the sum of the products of each remainder into all the divisors preceding the one that produced it, add the first remainder, and this sum will be the true remainder.

1. Divide 2486 by 105.

The factors of 105 are 3, 5 and 7.

OPERATION.

1	3)2486	
2.	5)828—2	1st remainder.
3.	7)165—3	2nd "
Quotient,	23—4	3rd "

EXPLANATION.—The small figures 1, 2, 3, on the left of the divisors are used to designate the numbers that have become dividends. The 3*d* remainder is the same as the 3*d* dividend, but a unit of the 3*d* dividend is equal to 5 units of the 2*nd* dividend; and a unit of the 2*nd* dividend is equal to 3 units of the 1*st* dividend, since the 1*st* and 2*nd* dividends have been divided respectively by 3 and 5. Therefore, a unit of the 3*d* remainder is equal to 5×3 units of the 1*st* remainder, and 4 units of the 3*d* remainder is $5\times3\times4=60$ units of the first remainder. For a similar reason 3 units of the 2*nd* remainder is equal to $3\times3=9$ units of the 1*st* remainder. To the sum of these products add the first remainder, and we have $60+9+2=71$ the true remainder.

2. Divide 4898 by 21.

3. Divide 9042 by 15.

4. Divide 11128 by 1155.

5. A man bought 15 horses for 2910 dollars; how much was that a piece?

6. If 27 barrels of flour cost 250 dollars, how much is that a barrel?

7. A wealthy merchant distributed 588 yards of cloth equally among 49 poor individuals; how many yards did they receive a piece?

8. A drover paid 1456 dollars for cattle, giving 56 dollars a head; how many cattle did he buy?

9. A farmer bought 98 acres of land for 2178 dollars; how much did he pay an acre?

10. In a certain corn-field there are 5229 hills of corn, and 63 rows; how many hills in a row?

ART. **49.** Division of abstract numbers, when the divisor, or dividend, or both have ciphers on the right.

1. Divide 82468524 by 24500.

OPERATION.

```
245|00)824685|24(3366 1524/24500
       735
       ---
        896
        735
        ----
        1618
        1470
        ----
         1485
         1470
         ----
Remainder, 1524
```

REMARK.—I cut off the ciphers on the right of the divisor, and as many places on the right of the dividend. After the division, affixing the remainder to the quotient with the divisor under it, and a horizontal line between them, and we have no remainder.

2. Divide 2468 by 10.
3. Divide 374232 by 100.
4. Divide 468324 by 1000.
5. Divide 36842 by 1100.
6. Divide 468234 by 450.
7. Divide 476324 by 4810.
8. Divide 846324 by 7800.
9. Divide 14786324 by 48300.
10. Divide 246832 by 470.
11. Divide 2476800 by 470.
12. Divide 8468300200 by 47600.
13. Divide 12468300200 by 3680.
14. Divide 4780024680000 by 8496000.
15. Divide 846847600847000 by 84000.

ABSTRACT EXAMPLES IN THE FUNDAMENTAL RULES.

ART. **50.** Quantities enclosed in a *parenthesis*, sometimes called the sign of *aggregation*, ( ), are to be subjected to the same operation Thus, $(3+6-2)\times5$, denotes

that the sum of 3 and 6, lacking 2 is to be multiplied by 5, the product of which is 35.

1. What is the value of the expression, $(465-27+140)\times 8$?

2. What is the value of the expression, $(846+47+96)\times 25$?

3. What is the value of the expression, $(897-47+86)\times 7-184$?

4. What is the value of the expression, $464+(843-87+9)\times 476-467$?

5. What is the value of the expression, $467-189+(88-14+215)\times 97$?

6. What is the value of the expression, $(462+7-146)\times(84-74+115)$?

7. What is the value of the expression, $96+(144-97)\times(86-47-189)\div 93$?

8. What is the value of the expression, $(47-23+12)\div 9+(98+4)\times(144-97)$?

9. What is the value of the expression, $(14+7)\times 2+(256-25)\div 21$?

10. What is the value of the expression, $(896-176)\div 144+(274+82)\times(86-47+8)$?

PRACTICAL QUESTIONS COMPRISING THE FOUR FUNDAMENTAL RULES.

1. Henry has 4684 dollars, which lacks 248 dollars of being 4 times James' fortune; and Jackson is worth 3 times as much as Henry and James together, lacking 3421 dollars. How much money have James and Jackson respectively?

2. A man bought an equal number of cows and horses for 9720 dollars; for the cows he gave 23 dollars a piece; and for the horses 97 dollars a piece; how many of each did he buy?

3. A merchant expended 336 dollars for an equal number of yards of broadcloth, consisting of three different kinds; the first, at 5 dollars a yard; the second, at 7 dol-

lars; and the third, at 9 dollars a yard. How many yards of each kind did he buy?

4. A farmer sold an equal number of chickens, ducks, and geese for 3540 cents; the chickens, at 12 cents each; the ducks, at 37 cents each; and the geese, at 69 cents each. How many of each kind did he sell?

5. A gave 1 eighth of 89648 dollars for a farm, which was 237 dollars more than it was worth; how much was the farm worth?

6. Light moves about 11550000 miles a minute; at this rate, how long would light be in passing from the sun to the earth, a distance of 95000000 of miles?

7. The product of two numbers is 91096; and one of the numbers is 472. What is the other number?

8. The quotient arising from dividing one number by another is 345; the dividend is 273585. What is the divisor?

9. The quotient arising from a certain division is 437; the divisor is 413; and the remainder 247. What is the dividend?

10. A farmer's yearly income was 19437 dollars. He paid for repairing his house 313 dollars; for hired help on his farm, 5 times as much, lacking 65 dollars; and for traveling expenses 2463 dollars. How much does he save yearly?

11. Bought 45 barrels of flour for 225 dollars; for what must it be sold a barrel to gain 135 dollars, and what will be the gain on each barrel?

12. Bought 130 acres of land for 5850 dollars; and sold 112 acres of it, at 75 dollars an acre, and the remainder for what it cost; how much did I gain by the bargain?

13. Bought 150 acres of land for 9750 dollars; and sold a part of it for 7140 dollars, at 85 dollars an acre;—how many acres had I remaining, and how much did I gain on every acre sold?

14. A farmer sold corn for 864 dollars; wheat for 895 dollars; rye and oats for 3 times as much as he received for the corn and wheat together, lacking 148 dollars. Out of these proceeds he bought 6 span of horses, at 275 dol-

lars a span; 5 yoke of oxen, at 125 a pair; and the remainder, lacking 738 dollars, he paid for land, at 65 dollars an acre. How many acres did he buy?

15. Bought 195 acres of land, at 84 dollars an acre, which cost 12 times as much as I paid for a span of fine horses. I have now 1468 dollars remaining. How much money had I at first?

16. If an army of 6000 men have provisions for 5 months, and 4400 men be disengaged; how long will the same provisions serve the remainder?

17. A certain tradesman can earn 54 dollars a month, but his necessary expenditures are 29 dollars a month. He desires to purchase a farm containing 75 acres, worth 35 dollars an acre. In what time can he save money enough to make the purchase?

18. Sold to my neighbor 12 cords of wood, at 5 dollars a cord; 65 barrels of corn, at 2 dollars a barrel; 45 head of cattle, at 65 dollars a head. In payment, I take 5 sacks of coffee, at 15 dollars a sack; 25 barrels of sugar, at 15 dollars a barrel; 2405 dollars in cash; and the remainder, in molasses, at 26 dollars a barrel. How many barrels of molasses ought I to receive?

19. A drover bought a certain number of cattle for 8050 dollars, and sold a certain number of them for 6237 dollars, at 63 dollars each, and gained on those he sold 1683 dollars; how many did he buy at first, and how much did he gain a piece on those he sold?

20. A speculator gave 18810 dollars for a certain number of acres of land, and sold a part of it for 7990 dollars, at 85 dollars an acre, and by so doing, lost 10 dollars on each acre; for how much must he sell the remainder an acre to gain 2180 dollars by the operation?

21. A farmer gave 37620 dollars for a farm, and sold a certain number of acres of it for 15980 dollars, at 85 dollars an acre, and by so doing lost 20 dollars an acre; for how much must he sell the remainder an acre to gain 4360 dollars by the operation?

CHAPTER III.

TABLES OF MONEY, WEIGHTS AND MEASURES.—ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF POLYNOMIALS, OR DENOMINATE NUMBERS.

ART. **51.** A SIMPLE NUMBER is either a unit, or a collection of units considered abstractly, that is, without reference to any particular thing; as 8, 16, 24, &c.

ART. **52.** A CONCRETE, or DENOMINATE NUMBER, is either a unit, or a collection of units having reference to some particular thing; as 4 feet, 5 dollars, 8 hours, 25 men, &c. The *measuring unit* of any quantity is a similar concrete unit, by means of which the quantity is expressed numerically.

ART. **53.** A MONOMIAL in Algebra, is a quantity of one term only; it may also, with propriety, be applied to an Arithmetical number, when it is expressed by a *single name* of a measuring unit; as, 5 dollars, 7 bushels, 10 men, &c.

ART. **54.** A POLYNOMIAL in Algebra is a quantity consisting of many terms; it may also be applied to denominate numbers, signifying a quantity of many names; as, 2 cwt. 3 qrs. 15 lbs., &c. It is, however, more generally applied to an abstract number consisting of many terms; as, $(4 + 6 + 8 + 9,)$ &c.

TABLE OF UNITED STATES CURRENCY.

10 Mills	make	1 Cent,	marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$.
10 Dollars	"	1 Eagle,	"	E.

ART. **55.** It will be observed that the measuring units in the United States currency increase in a TENFOLD ratio, as in abstract numbers. Hence this currency will be treated of under Decimal Fractions. The measuring units of other kinds of quantity, increase from lower to higher

orders according to the scales of increase given in the following tables.

ENGLISH OR STERLING MONEY.

ART. **56.** ENGLISH MONEY is the currency of England. Its denominations are Pounds, Shillings, Pence, and Farthings.

TABLE.

4 Farthings (far. or qr.)	make	1 Penny,	marked	d.
12 Pence	"	1 Shilling,	"	s.
20 Shillings	"	1 Pound,	"	£.

TROY WEIGHT.

ART. **57.** By this weight are weighed gold, silver, and jewels.

REMARK.—The original of all weights used in England, was a grain of wheat, taken from the middle of the ear ; 32 of these, dried, were to make 1 pennyweight.

Since then it was agreed to divide the same pennyweight into 24 equal parts, still called *grains*, being the least weight in common use.

TABLE.

24 Grains (gr.)	make	1 Pennyweight,	marked,	pwt.
20 Pennyweights	"	1 Ounce,	"	oz.
12 Ounces	"	1 Pound,	"	lb.

AVOIRDUPOIS WEIGHT.

ART. **58.** Avoirdupois Weight is used to weigh all things of a course nature, as groceries, some liquids, and all metals, except gold and silver.

TABLE.

16 Drams (dr.)	make	1 Ounce,	marked	oz.
16 Ounces	"	1 Pound,	"	lb.
25 Pounds*	"	1 Quarter,	"	qr.
4 Quarters	"	1 Hundred Weight,	"	cwt.
20 Hundred Weight	"	1 Ton,	"	T.

* NOTE.—In buying and selling articles, it is customary to call 25 pounds, 1 qr., instead of 28; and 100 pounds, 1 cwt., instead of 112 pounds, as was formerly done.

Apothecaries' Weight.

Art. **59.** Apothecaries' Weight is used in compounding, or weighing small quantities of medicines, as for prescriptions. But medicines and drugs by the quantity, are generally bought and sold by avoirdupois weight. The pound and ounce Apothecaries' Weight equals the pound and ounce Troy Weight.

TABLE.

20 Grains (gr.)	make	1 Scruple,	marked	℈
3 Scruples	"	1 Dram,	"	ʒ
8 Drams	"	1 Ounce,	"	℥
12 Ounces	"	1 Pound,	"	℔

Cloth Measure.

Art. **60.** Cloth Measure is used in measuring cloth, lace, ribbons, and all other articles sold by the yard.

TABLE.

$2\frac{1}{4}$ Inches (in.)	make	1 Nail,	marked	na.
4 Nails, or 9 in.	"	1 Quarter of a yard,	"	qr.
4 Quarters	"	1 Yard,	"	yd.
3 Quarters	"	1 Ell Flemish,	"	E. Fl.
5 Quarters	"	1 Ell English,	"	E. E.
6 Quarters	"	1 Ell French,	"	E. Fr.

Long Measure.

Art. **61.** This measure is used in measuring distances.

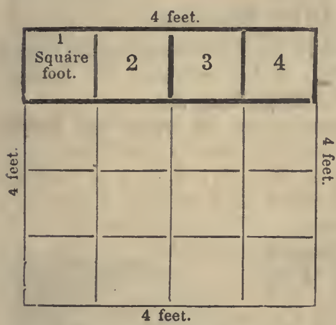
TABLE.

12 Inches (in.)	make	1 Foot,	marked	ft.
3 Feet	"	1 Yard,	"	yd.
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet,	"	1 Rod, Pole, or Perch,	"	rd.
40 Rods	"	1 Furlong,	"	fur.
8 Furlongs	"	1 Mile,	"	m
3 Miles	"	1 League,	"	lea
60 Geographic miles, or $69\frac{1}{2}$ statute or league miles	"	1 Degree,	"	deg. or °.
360 Degrees	"	1 Circle,	"	circ.

4 Inches	make 1 Hand,	Used in measuring the height of horses.
6 Feet	" 1 Fathum,	Used in measuring depths at sea.

Superficial, or Square Measure.

Art. 62. This measure is used for measuring all kinds of surfaces, such as land, boards, plastering, and everything else, in which length and breadth only are considered.



A Square is a figure having *four equal* sides, and four equal angles, or four right angles.

This diagram is called *four feet square*, as it is four feet each way. Each of the small squares, (within the large square,) represents 1 *square foot.* There are 4 square feet in each row, and 4 rows in the whole square; therefore, there are 4 times 4 square feet, equal to 16 *square feet*, in 4 feet square; hence there is a difference of 12 *square* feet between 4 feet square, and 4 square feet. The 4 square feet is represented by the squares 1, 2, 3, and 4; and the 4 feet square, by the large square which contains 16 square feet. From the above we infer that the superficial contents of a square, or any rectangular figure is found by *multiplying its length with its width.*

TABLE.

144 Square Inches (sq.in.)	make	1 Square Foot,	marked	sq. ft.
9 Square Feet	"	1 Square Yard,	"	sq. yd.
$30\frac{1}{4}$ Square Yards	"	1 Sq. Rod, or Pole,	"	P.
40 Square Rods or Poles	"	1 Rood,	"	R.
4 Roods	"	1 Acre,	"	A.
640 Acres	"	1 Square mile,	"	S. M.

Surveyor's Measure.

In measuring land, roads, &c., Gunter's chain is used; the length of which is 4 rods, or 66 feet.

TABLE.

$7\frac{92}{100}$ Inches (in.)	make	1 Link	marked	li.
25 Links	"	1 Rod, or Pole,	"	P.
4 Poles, or 100 links	"	1 Chain,	"	cha.
10 Chains	"	1 Furlong,	"	fur.
8 Furlongs, or 80 chains,	"	1 Mile,	"	M.
10 Square Chains	"	1 Acre,	"	A.

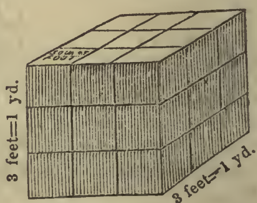
Solid, or Cubic Measure.

Art. 63. This measure is used in measuring all things that have length, breadth, and thickness; as timber, boxes of goods, capacity of ships, &c., &c.

A cube is a solid, bounded by six equal and square sides.

If each of the sides of a cube is 1 foot it is called a *cubic foot.* If each of the sides of a cube be 3 feet = 1 yard, it is called a *cubic yard.*

The annexed diagram represents a cubic yard. Since each of the sides of a cubic yard is 3 feet each way; each of these sides will contain 9 square feet. If from one side of this cube we cut off a piece 1 foot in thickness, we evidently have 9 *solid feet;* and as the whole block is 3 feet thick, it must contain 3 times 9 = 27 *solid feet.* Hence, to find the solid contents of a cube, we *multiply its length, breadth, and thickness together.*



3 feet=1 yd.

TABLE.

1728 Cubic inches (cu. in.)	make	1 Cubic foot,	marked	cu. ft.
27 " feet	"	1 " yard,	"	cu. yd
40 " feet	"	1 Ton,	"	T.
16 " feet	"	1 Cord foot,	"	c. ft
8 Cord feet, or 128 Cubic feet	"	1 Cord of wood,	"	C.

Wine Measure.

Art. **64.** By this measure all liquids, except beer are measured.

The wine gallon contains 231 cubic inches.

TABLE.

4	Gills (gi.)	make	1 Pint,	marked	pt.
2	Pints	"	1 Qnart,	"	qt.
4	Quarts	"	1 Gallon,	"	gal.
$31\frac{1}{2}$	Gallons	"	1 Barrel,	"	bar.
42	Gallons	"	1 Tierce,	"	tier.
63	Gallons	"	1 Hogshead,	"	hhd.
2	Hogsheads	"	1 Pipe,	"	pi.
2	Pipes	"	1 Tun,	"	tun.

Ale, or Beer Measure.

Art. **65.** By this measure ale, beer, and milk, are measured.

The beer gallon contains 282 cubic inches.

TABLE.

2	Pints (pt.)	make	1 Quart,	marked	qt.
4	Quarts	"	1 Gallon,	"	gal.
36	Gallons	"	1 Barrel,	"	bar.
$1\frac{1}{2}$	Barrels, or 54 Gallons	"	1 Hogshead,	"	hhd.

Dry Measure.

Art. **66.** Grain, salt, coal, &c., are measured by Dry Measure.

The Dry Gallon contains $268\frac{4}{5}$ cubic inches. The Winchester Bushel contains $2150\frac{2}{5}\frac{1}{0}$ cubic inches. A Cylindrical Measure, 8 inches deep and $18\frac{1}{2}$ inches in diameter, contains 1 bushel.

TABLE.

2 Pints (pt.)	make	1 Quart,	marked	qt.
8 Quarts	"	1 Peck,	"	pk.
4 Pecks	"	1 Bushel,	"	bu.
36 Bushels	"	1 Chaldron	"	ch.
32 Bushels	"	1 Chaldron in the United States.		

CIRCULAR MEASURE.

ART. **67.** This measure is used in noting any part of the circumference of a circle; it is also used in reckoning *latitude* and *longitude*, and the revolutions of the heavenly bodies.

TABLE.

60 Seconds (″)	make	1 Minute,	marked	′
60 Minutes	"	1 Degree,	"	°
30 Degrees	"	1 Sign,	"	S.
12 Signs, or 360 Degrees	"	1 Circle,	"	C.

MEASURE OF TIME.

ART. **68.** This measure is applied to the divisions and subdivisions of time.

TABLE.

60 Seconds (sec.)	make	1 Minute,	marked	min.
60 Minutes	"	1 Hour,	"	hr.
24 Hours	"	1 Day,	"	da.
7 Days	"	1 Week,	"	wk.
4 Weeks	"	1 Month,	"	mo.
12 Calendar months, or 52 Weeks, 1 day, and 6 hours	"	1 Year,	"	yr.
365 Days, 6 hours, (nearly,)	"	1 Year,	"	yr.

The *Solar* year consists of 365 days, 5 hours, 48 minutes, and $51\frac{3}{5}$ seconds, and is the exact time in which the Earth performs one revolution around the Sun.

The *Civil* year consists of 365 days. Hence, the difference between the Solar and the Civil year is nearly 6 hours, making about 1 day in 4 years.

As the difference between the Solar and the Civil year confused dates, Julius Cæsar made the first correction of the calender by introducing an *intercalerary* day in every *fourth* year. This day was added to the month of February making it to consist of 29 instead of 28 days. This fourth year was denominated *Bissextile*, and is now usually called *Leap Year*. As the correction should have

been 5 hrs. 48 min. and $51\frac{3}{5}$ sec., instead of 6 hours, by considering every fourth year as consisting of 366 days, there was involved an error amounting to about 18 hours in every 100 years; to remedy which every one-hundredth year was considered as having only 365 days. But the allowance of a *whole* day in every 100 years was too much, by nearly *one-fourth* of a day, which excess in every 400 years amounted to an *entire day.*

Hence, every year, (except the centennial years,) that is divisible by 4 is a *Leap Year*, and every centennial year that is divisible by 400 is also, a *Leap Year*. The next centennial year that will be a Leap Year is 2000.

The following are the names of the twelve calendar months, which compose the civil year, and the number of days in each:—

			Names.		Days.
Winter.	1st month		January,	— — —	31
	2d	"	February,	— — —	28—in leap year 29.
Spring.	3d	"	March,	— — —	31
	4th	"	April,	— — —	30
	5th	"	May,	— — —	31
Summer	6th	"	June,	— — —	30
	7th	"	July,	— — —	31
	8th	"	August,	— — —	31
Autumn	9th	"	September,	— — —	30
	10th	"	October,	— — —	31
	11th	"	November,	— — —	30
Winter.	12th	"	December,	— — —	31

"Thirty days hath September,
April, June, and November;
February 28, alone,
All the rest have thirty-one,
Except in Leap Year; then is the time
When February has twenty-nine."

The following Table will enable us readily to determine the number of days from one date, to any other particular date in the same year:—

TABLE,

EXHIBITING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY.											
	Jan.	Feb.	Mar.	April	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY,....	355	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY,..	334	365	28	59	89	120	150	181	212	242	273	303
MARCH......	306	337	365	31	61	92	122	153	184	214	245	275
APRIL,......	275	306	334	365	30	61	91	122	153	183	214	244
MAY,........	245	276	304	335	365	31	61	92	123	153	184	214
JUNE,.......	214	245	273	304	344	365	30	61	92	122	153	183
JULY,.......	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST, ...	153	184	212	243	273	304	334	365	31	62	92	122
SEPTEMBER,..	122	153	181	212	242	273	303	334	365	30	61	91
OCTOBER.....	92	123	151	182	212	243	273	304	335	365	31	61
NOVEMBER,..	61	92	120	151	181	212	242	273	304	334	365	30
DECEMBER,...	31	62	90	121	151	182	212	243	274	304	335	365

How many days from the 12th of May to the 12th of October? In the column of months on the left of the page we find April;—passing the eye along the horizontal row of figures till it comes to the perpendicular column, headed "Oct.," we find 153 days to be the time. If, instead of "12th of Oct.," in the above question, we substitute the 20th of Oct., then to the 153 days, add the excess of 20 above, $12 = 8$; and we have $153 + 8 = 161$ days, for the number of days.

When there are 29 days in February the proper allowance must be made, as the table considers 28 days in February.

Books.

A sheet folded in	two	leaves	is called	a *Folio.*
" " " "	four	"	"	" a *Quarto* or 4to.
" " " "	eight	"	"	" an *Octavo*, or 8vo.
" " " "	twelve	"	"	" a *Duodecimo* or 12mo
" " " "	eighteeen	"	"	" an 18mo.
" " " "	twenty-four	"	"	" a 24mo.

MISCELLANEOUS TABLE.

12 Units	make	1 Dozen.
12 Dozen	"	1 Gross.
12 Gross	"	1 Great Gross.
20 Units	"	1 Score.

24 Sheets of paper	make	1 Quire.
20 Quires	"	1 Ream.

30 Pounds	make	1 Bushel of oats.
46 Pounds	"	1 Bush. of buckwheat or barley.
56 Pounds	"	1 Bushel of Indian corn or rye.
60 Pounds	"	1 Bushel of wheat.
70 Pounds	"	1 Bushel of salt.

55 Pounds	make	1 Firkin of butter.
196 Pounds	"	1 Barrel of flour.
200 Pounds	"	1 Barrel of pork.
200 Pounds	"	1 Barrel of beef.
200 Pounds	"	1 Barrel of shad or salmon.

14 Pounds of lead, or iron	make	1 Stone.
$21\frac{1}{2}$ Stone	"	1 Pig.
8 Pigs	"	1 Fother.

ADDITION OF DENOMINATE NUMBERS.

ART. **69.** ADDITION *of denominate numbers* is finding the sum of two or more numbers of different denominations in the *same kind* of measure.

1. What is the sum of £8 16s. 5d., £3 12s. 9d., £6 15s. 10d., and £4 10s. 3d.?

OPERATION.

	20	12
£	s.	d.
8	16	5
3	12	9
6	15	10
4	10	3
23	15	3

EXPLANATION.—Write the given numbers of the same denomination one under another, and place 12 over the d. and 20 over the s. (to cause the pupil to bear in mind that 12 pence make 1 shilling, and 20 shillings make 1 pound.) Then, commencing at the column of pence, we add it as in simple addition, and obtain for the sum 27 pence. In 27 pence, how many shillings? There are 12 pence in 1s., therefore, one-twelfth of the number of pence equals the number of shillings;—12 is contained in 27, 2 times and 3d. remaining. Place the 3d. under the column of pence, and add the 2s. to the column of shillings, and we obtain for

the sum, 55 shillings. In 55s., how many pounds? There are 20s. in £1, therefore, one-twentieth of the number of shillings, equals the number of pounds;—20 is contained in 55, 2 times and 15s. remaining. Place the 15s. under the column of shillings, and add the £2, to the column of pounds, and we obtain for the sum £23. This we place under the column of £, and we have for the answer, £23 15s. 3d.

2.

£	s.	d.	far.
14	10	6	2
16	15	10	3
13	12	4	1
23	9	3	0

3.

£	s.	d.	far.
16	11	4	1
23	13	11	2
25	15	8	0
17	18	10	3

Troy Weight.

4.

lb.	oz.	pwt.	gr.
24	7	15	20
30	9	19	13
22	8	17	14
16	11	0	12

5.

lb.	oz.	Pwt.	gr.
14	7	14	20
24	9	16	16
26	10	3	18
36	11	17	15

Avoirdupois Weight.

6.

T.	cwt.	qr.	lb.	oz.	dr.
17	14	3	20	8	9
2	8	1	15	10	7
4	13	0	13	6	10
6	16	2	12	11	8

7.

T.	cwt.	qr.	lb.	oz.	dr.
25	14	3	22	11	2
35	19	1	21	10	8
16	20	0	16	9	7
17	21	1	19	1	9

Apothecaries' Weight.

8.

lb.	℥.	ʒ.	℈.	gr.
5	1	2	1	15
8	10	7	2	19
14	10	2	3	7
5	1	2	3	16
7	9	3	1	12

9.

lb.	℥.	ʒ.	℈.	gr.
10	1	6	0	10
0	1	2	1	16
13	2	7	3	20
11	0	5	4	13
17	2	4	2	18

CLOTH MEASURE.

10.

yd.	qr.	na.
5	3	3
8	0	1
7	2	2
8	3	0
1	3	1

11.

E.Fr.	qr.	na.
3	4	3
7	5	0
10	2	1
19	3	2
23	1	1

12.

E.E	qr.	na.
17	4	2
18	3	1
24	1	3
17	2	0
15	0	2

LONG MEASURE.

13.

m.	fur.	rd.	yd.	ft.	in
91	7	29	4	2	10
13	3	27	0	1	9
7	7	39	1	2	4
15	6	32	0	0	7
23	7	23	1	2	8
30	5	28	0	1	6

14.

deg.	m.	fur.	rd.	yd.	ft.	in.
122	19	4	1	5	0	9
12	7	3	23	0	1	9
27	4	7	36	1	2	11
32	1	3	26	0	0	10
34	0	6	39	1	2	8
17	4	5	37	0	1	7

SUPERFICIAL, OR SQUARE MEASURE.

15.

S. M.	A.	R.	P.	sq.yd.	sq. ft.	sq. in.
123	465	1	27	20	6	14
12	121	1	12	8	3	122
36	376	2	32	7	7	100
37	468	1	17	2	6	132
19	478	3	12	0	8	140
17	300	0	34	0	5	96

SURVEYOR'S MEASURE.

16.

m.	fur.	cha.	P.	li.
80	2	4	2	11
18	6	3	1	21
13	2	9	3	17
22	7	8	2	18
13	5	7	1	16
11	3	5	0	14

17.

m.	fur.	cha.	P.	li.
187	2	2	1	9
11	4	9	3	17
24	3	8	1	12
36	1	7	0	14
41	0	0	1	18
73	7	6	1	23

Solid, or Cubic Measure.

18.

T.	cu. ft.	cu. in.
84	12	1364
18	32	1431
16	12	931
30	28	1246
73	17	863
96	14	1241

19.

C.	cu. ft.	cu. in.
12	120	1463
12	121	1612
91	112	1316
36	97	362
97	88	1473
12	12	784

Wine Measure.

20.

Tun.	hhd.	gal.	qt.	pt.	gi.
94	1	26	2	1	0
4	1	21	3	1	3
12	0	32	1	0	2
17	1	47	2	1	1
26	0	53	1	0	0
34	0	60	1	1	2

21.

hhd.	gal.	qt.	pt.
100	19	1	1
12	37	3	1
32	46	0	0
14	37	2	1
18	17	1	1
22	6	2	0

Ale, or Beer Measure.

22.

hhd.	gal.	qt.	pt.
39	31	1	0
3	23	2	1
8	40	3	0
7	37	1	1
6	38	2	1
12	52	3	1

23.

bar.	gal.	qt.	pt.
49	1	3	1
7	32	3	1
6	27	1	0
7	13	2	0
12	18	3	1
14	17	1	1

Dry Measure

24.

ch.	bu.	pk.	qt.	pt.
75	1	2	2	1
12	35	3	7	1
16	25	1	5	0
10	32	2	4	0
11	17	1	6	1
22	34	0	3	1

25.

bu.	pk.	qt.	pt.
81	0	0	1
23	3	7	1
12	0	1	0
17	2	6	0
10	3	5	1
16	1	4	1

CIRCULAR MEASURE.

26.

C.	S.	°	'	"
21	0	15	38	5
1	7	12	40	32
2	9	17	35	16
3	8	23	37	46
3	10	29	57	54
8	11	21	46	37

27.

C.	S.	°	'	"
17	11	19	53	11
1	3	21	39	52
2	9	13	42	47
7	6	27	55	19
1	11	28	18	17
4	3	18	16	56

MEASURE OF TIME.

28.

yr.	da.	hr.	min.	sec.
99	155	1	50	44
12	10	13	42	27
16	102	18	24	36
19	8	21	54	57
23	13	19	49	48
29	18	23	58	56

29.

wk.	da.	hr.	min.	sec.
49	3	19	36	5
3	4	20	42	17
1	6	21	47	39
2	2	19	58	52
17	4	16	48	58
23	5	12	18	19

SUBTRACTION OF DENOMINATE NUMBERS.

ART. 70. SUBTRACTION *of denominate numbers* is finding the difference between two denominate numbers.

1. From £38 8s. 10d. 1 far., take £12 15s. 4d. 3 far.

OPERATION.

		20	12	4
	£.	s.	d.	far.
Min.	38	8	10	1
Sub.	12	15	4	3
Rem.	25	13	5	2

EXPLANATION.—Write the subtrahend under the minuend, observing to place the numbers of the same denomination one under another; and begin at the right to subtract. We cannot take 3 farthings from 1 far., therefore, from the 10d. (of the minuend) we take 1 penny, which equals 4 far., and add it to 1 far., which makes it 5 farthings; 3 far. from 5 far. leave 2 far., which place under the column of farthings. We now take the 4d. from the 9d., or add 1 penny to the 4d., and take this sum from 10d., which in either case gives the same remainder, 5d. As we cannot take 15s. from 8s., we borrow £1, = 20s., from the £38, and add it to the 8s.; from this sum, we take the 15s., and obtain 13s. for a rema nder. We now take £12 from £37,

or add £1 to the £12, and take the sum from the £38, which in either case, gives the same remainder, £25. Hence the difference of the two quantities is £25 13s. 5d. 2 far.

2.

£	s.	d.	far.
24	8	6	2
16	12	7	3

3.

lb.	oz.	pwt.	gr.
25	8	17	21
14	7	18	23

4.

T.	cwt.	qr.	lb.	oz.	dr.
50	16	1	23	10	12
27	7	3	24	3	14

5.

T.	cwt.	qr.	lb.	oz.	dr.
14	10	2	12	4	8
5	14	3	20	7	12

6.

lb.	℥.	ʒ.	℈.	gr.
14	11	6	1	12
12	10	3	2	15

7.

yd.	qr.	na
18	2	1
12	3	3

8.

E.Fr.	qr.	na.
14	4	2
10	5	3

9.

E.E.	qr.	na
19	2	1
16	4	3

10.

leg.	m.	fur.	rd.	yd.	ft.	in
21	2	7	21	3	1	8
18	45	3	25	4	2	10

11.

m.	fur.	cha.	P.	lin.
16	5	3	1	10
13	2	8	2	12

12.

S.M.	A.	R.	P.	sq. yd.
36	276	2	12	3
24	108	3	37	$25\frac{1}{4}$

13.

T.	cu.ft.	cu.in.
16	32	1421
6	37	1675

14.

C.	cu ft.	cu.in.
16	110	1612
11	116	1719

15.

Tun.	hhd.	gal.	qt.	pt.	gi.
32	0	24	3	0	2
15	2	52	3	1	3

16.

hhd.	gal.	qt.	pt
62	41	0	0
49	60	2	1

17.

ch.	bu.	pk.	qt.	pt.
30	12	3	3	1
17	30	3	7	0

18.

bu.	pk.	qt.	pt.
27	2	6	0
9	3	7	1

19.

C.	S.	°	′	″
20	10	15	24	32
9	5	24	56	52

20.

wk.	da.	hr.	min	sec.
36	3	12	43	15
26	6	20	55	32

PRACTICAL QUESTIONS IN ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS.

1. From a piece of cloth containing 27 yards 3 qrs. 1 na., there were taken three garments; the first contained 3 yds. 3 qrs. 2 nas.; the second 4 yds. 1 qr. 3 nas.; and the third 2 yds. 3 qrs. 3 nas.;—how much remained?

2. Bought a hogshead of sugar wieghing 9 cwt. 3 qrs. 21 lbs.; sold to A 1 cwt. 2 qrs. 15 lbs.; to B 2 cwt. 3 qrs. 24 lbs.; and to C 3 cwt. 1 qr. 15 lbs.;—how much remained unsold?

3. A man agrees to build 136 rods and 15 feet of stone fence;—at one time he built 36 rds. 2 feet; at another time 56 rds. 3 feet; and at another time 10 rds. 1 foot. How much still remains to be built.

4. I agreed to let a person have 24 T. 9 cwt. 2 qrs. 15 lbs. of hay. He took away four loads, the weight of which were as follows: the first weighed 16 cwt. 2 qrs. 18 lbs.; the second, 19 cwt. 3 qrs. 12 lbs.; the third, 1 T. 2 cwt. 1 qr. 21 lbs.; and the fourth, 1 T. 5 cwt. 2 qrs. 14 lbs.;—to how much hay is he still entitled?

5. How many yard of cloth in three pieces: the first containing 12 yds. 3 qrs. 2 nas.; the second 6 E. English 2 qrs. 1 na.; the third 9 E. French, 1 qr. 3 nas.?

6. Bought three pieces of cloth: the first containing 25 yds. 3 qrs. 1 na.; the second 47 yds. 1 qr. 3 nas.; and the third 35 yds. 3 qrs. 2 nas.;—I sold 73 yds. 3 qrs. 2 nas. of it. How much remained unsold?

7. A merchant bought, at one time 956 bushels and 3 pecks of Indian corn; at another time 759 bushels, 2 pks. and 7 quarts; and sold 325 bush. 3 pks. and 6 qts. of it. How much had he remaining?

8. John is 23 years, 9 month, and 18 days old; James is 18 years, 10 months, and 25 days old. What is the difference of their ages?

9. Suppose a person was born February 29, 1788; how many birth-days will he have seen on February 29, 1840, not counting the day on which he was born?

10. A merchant sold goods to the amount of £397 18s. 6d. 2 qrs.; and received in payment £199 19s. 10d. 3 qrs.; how much remains due?

11. From a pile of wood containing 423 cords, I sold at one time, 56 C. 112 cu. ft.; at another time, 97 C. 113 cu. ft.; at another time, 126 C. 96 cu. ft. How many cords remain unsold?

12. How long from the birth of William Shakspeare, April 23, 1564, to the birth of Milton, Dec. 9, 1608?

13. A farmer raises 125 bush. 2 pks. 6 qts. of wheat on one field; 197 bush. 1 pk. 7 qts. on another field: he sells to one person 97 bush. 3 pks. 7 qts.; and to another person 112 bush. 2 pks. 6 qts. How many bushels has he remaining?

14. A gentleman owned three tracts of land: the first of which contained 127 acres, 3 R. 15 rods; the second, 496 A. 1 R. 25 rods; the third, 525 A. 0 R. 35 rods; how much remained after he sold 1008 A. 2 R. 25 rods?

15. Suppose a note given Sept. 10, 1796, to be paid March 5, 1808. How long was the note on interest, if we count 30 days to the month? How long if the time is accurately computed?

Multiplication of Denominate Numbers.

Art. **71.** Multiplication *of denominate numbers* is taking a quantity of different denominations as many times as there are units in another number.

Multiply £5 12s. 6d. by 5.

OPERATION.		
£	s.	d.
5	12	6
		5
28	2	6

EXPLANATION.—The numbers being properly written down, we begin at the right to multiply. 5 times 6d. are 30d., in 30d. how many shillings? There are 12d. in 1s., therefore, one-twelfth of the number of pence equals the number of shillings. 12 is contained in 30, 2 times, and 6d. remaining;—write the 6d. under pence, and reserve the 2s. 5 times 12s. are 60s., and 2s. added, are 62s., which equals £3 2s.;—write the 2s. under shillings, and reserve the £3. 5 times £5 are £25, and £3 added are £28. Hence, &c.

2.

£	s.	d.	far.
12	10	8	3
			4

3.

cwt.	qr.	lb.	oz.
22	3	21	12
			5

4.

T.	cwt.	qrs.	lb.	oz.
4	12	1	20	12
				7

5.

m.	fur.	rd.	ft.
12	7	32	2
			12

6.

deg.	m.	fur	rd.
18	21	4	20
			8

7.

yds.	qrs.	nas
14	3	2
		14

8.

yds.	qrs.	na.
17	3	1
		20

9.

cwt.	qr.	lb.
18	3	23
		15

10.

T.	cwt.	qr.	lb.	oz.	dr.
4	17	3	21	12	14
					9

11. How much cloth will it take for 9 suits of clothes, if each suit require 8 yds. 2 qrs. 2 nas.?

12. How long will it take a man to chop 14 cords of wood, if it take him 7 hours, 40 minutes, and 50 seconds to chop 1 cord?

13. What is the weight of 12 hogsheads of sugar, each weighing 8 cwt. 3 qrs. 23 lb.?

14. If a span of horses, at 1 load, can draw 1 cord 212 cubic feet of wood, how many cords can they draw in 14 loads?

15. If a family of 6 persons, consume 10 gallons, 3 quarts, and 1 pint of molasses in 1 week; what quantity will a family of double the number of persons consume in 1 year?

16. What is the weight of 18 silver spoons, if each weigh 5 oz. 14 pwt. 20 grs.?

17. If 1 acre of land produce 45 bush. 3 pks. 7 qts. 1 pt. of wheat, how much will 12 acres produce?

18. If a man walk 25 miles, 5 fur. 27 rds. in 1 day, how far can he walk in 9 weeks, not counting Sunday?

19. An estate of £3295 15s. 6d. is divided among four children: the first has £725 16s. 11d.; the second twice as much, lacking £802 18s. 9d.; the third £846 17s. 9d.; and the fourth the remainder. How much did the fourth receive?

20. If a locomotive move 1 m. 25 rds., in 1 minute; how far will it move in 1 day?

Division of Denominate Numbers.

Art. **72.** Division *of denominate numbers* is the process of finding any proposed part of a given number, composed of two or more denominations of the same kind of measure.

1. If 5 barrels of sugar weigh 9 cwt. 1 qr. 10 lbs., how much will 1 barrel weigh?

OPERATION.

	cwt.	qr.	lb.
5)	9	1	10
	1	3	12

Explanation.—Write the divisor on the left of the dividend, as in division of abstract numbers 5 is contained in 9, *once* and 4 cwt. remaining. 4 cwt. = 16 qrs., to which add the 1 qr. and it equals 17 qrs., 5 is contained in 17, 3 times, and 2 qrs. remaining. 2 qrs. = 50 lbs., to which add the 10 lbs., and it = 60 lbs. 5 is contained in 60, 12 times. Therefore, one-fifth of 9 cwt. 1 qr. 10 lbs., is 1 cwt. 3 qrs. 12 lbs.

Note.—It is impossible to divide one concrete number by another, (See Art. 44) hence in the above example we do not divide 9 cwt. 1 qr. 10 lbs. by 5 barrels, but we separate the 9 cwt. 1 qr. 10 lbs. into 5 equal parts; the 5 barrels, being considered an *abstract* number.

2.

	£	s.	d.	far.
)	62	7	9	3

3.

	cwt.	qr.	lb.	oz.
6)	101	1	13	8

4.

	T.	cwt.	qr.	lb.
7)	32	14	0	15

5.

	m.	fur.	rd.	ft.
12)	118	1	0	12

5.

	deg.	m.	fur.	rd.
9)	144	26	4	25

7.

T.	cwt.	qr.	lb.	oz.	dr.
9)44	1	1	1	3	14

8.

yds.	ft.	in.
7)196	2	11

9.

A.	R.	P.
11)346	3	37

10.

T.	cwt.	qr.	lb.	oz.	dr.
5)19	18	3	20	12	13

11. Divide, £346 18s. 4d. 2 far. by 47.

OPERATION.

```
         20   12   4
    £    s.   d.   far.
47)346   18   4    2   (£7 7s. 7d. 2 far.  Ans.
   329
   ---
    17
    20
   ---
 47)358(7s.
    329
    ---
     29
     12
    ---
47)352(7d.
   329
   ---
    23
     4
    ---
 47)94(2 far.
    94
    --
     0
```

12. Divide 137 lbs. 9 oz. 18 pint. 19 grs. by 23.

13. If 451 individuals share equally 8021 T. 12 cwt. 1 qr. 6 lbs. 8 oz. of sugar, how much will each receive?

14. If 13 hogsheads of sugar weigh 6 T. 8 cwt. 2 qrs. 7 lbs.; how much will 1 hogshead weigh?

15. A vintner sold 33 hhds. 56 gals. of wine, to 15 different men; how much did each buy, providing they each purchased an equal quantity?

16. If a man travel 348 mi. 1 fur. 12 rds. in 28 days, how far, on an average is that a day?

17. A merchant sold 320 yds. 2 qrs. 2 nas. of broadcloth, in 19 successive days; how much did he sell daily, providing he sold the same quantity each day?

18. A produce dealer divided 132 bushels, 3 pks. 7 qts. of wheat, equally among 23 of his poor neighbors; how much did each receive?

19. 26 men bought 645 acres 20 P. of land, and are to share it equally; how much ought each to receive?

20. A speculator bought 719 cwt. 1 qrs. 3 lb. of sugar, and sold it to 36 men; how much did each receive, providing each bought the same quantity?

Practical Questions combining Addition, Subtraction, Multiplication and Division of Denominate Numbers.

1. A farmer having 19 cwt. 2 qrs. 19 lbs. of pork, sold 5 cwt. 3 qrs. 1 lb. of it, and the remainder he put into 6 barrels; how much did each barrel contain?

2. Bought of A 97 acres, 2 R. 12 P. of land, of B 4 times as much, lacking 7 acres, 1 R., and of C one-half as much as of A and B together; how much did I buy of B and C respectively, and how much in all?

3. A merchant bought 9 pieces of silk, each containing 57 yds. 3 qrs. Having sold to another merchant one-third of it, and to 4 ladies, each 9 yds. 3 qrs. 3 nas., how much remains unsold?

4. A farmer has three fields of wheat: from the first he obtains 224 bus. 2 pks. 2 qts.; from the second one-half as much, increased by 76 bus. 3 pks. 1 qt.; and from the third, as much as from the other two, lacking 84 bus. 2 pks. 7 qts. How much did he obtain from the three fields?

5. From one-half of a piece of cloth containing 82 yds. 2 qrs., a tailor cut six suits of clothes. How much did each suit contain?

6. A, B, C, and D, having 4 cwt. 3 grs. 20 lbs. of sugar, agree to divide it as follows: A is to take 15 lbs

and one-fifth of the remainder; B 1 qr. 3 lbs. and one-fourth of the remainder; C 1 qr. 12 lbs. and one-third of the remainder; and D is to have what now remains. How much sugar should each receive?

7. A, B, C, and D, share 840 bushels, 3 pks. of wheat as follows: A takes 16 bush. 3 pks. and one-fourth of the remainder; B takes 14 bush. 2 pks. and one-third of what remains; C takes 13 bush. $2\frac{1}{3}$ pks. and one-half of what remains; and D takes what now remains. How much does each receive?

8. Bought of one man 8 bus. 2 pks. 3 qrs. of grass-seed; of another man 3 times as much, and 2 bus. 2 qts. more; and of another 3 times as much as of the second, lacking 1 bus. 1 pk. 6 qts. How much did I buy of each respectively, and how much of all?

9. 32 men agree to construct 28 miles. 4 fur. 32 rds. of road;—after completing one-half of it, one-fourth of the number of men left the company. What distance did each man construct before and after one-fourth of the men left?

10. A, B, C, and D, having 184 bus. 2 pks. of wheat, agree to divide it as follows: A is to have one-half of the whole; B is to have one-third of the remainder; C is to have one-fourth of what then remains; and D is to have what is left. What is the portion of each?

11. Divide 448 acres, 3 R. 24 P. of land among A, B, C, and D, so that A shall have one-eighth of the whole, + 4 acres, 3 rds. 6 pls.; B one-fifth of the remainder; C one-third of what then remains; and D the rest. How much will each one have?

12. An estate of £2490 is to be divided among a widow, two sons, and three daughters;—the widow receives one-third of the whole, lacking £346; the youngest son receives as much as the widow, + £212; the oldest son receives as much as the widow and youngest son together, lacking £335 10s.; and the three daughters share equally of the remainder. How much does each receive?

REDUCTION.

ART. **73.** If the quantity is to be changed from a *higher* to a *lower* denomination, the process is called REDUCTION DESCENDING;—if from a *lower* to a *higher* denomination, REDUCTION ASCENDING.

REDUCTION DESCENDING.

1. In £34 15*s.* 6*d.* how many pence?

OPERATION.

	20	12
£	s.	d.
34	15	6
20		
695		
12		
8346		

EXPLANATION.—There are 20s. in £1; therefore, 20 times the number of pounds equal the number of shillings. 20 times 34 are 680, and 15s. added = 695s. There are 12d. in 1s.; therefore, 12 times the number of shillings equal the number of pence. 12 times 695 are 8340, and 6d. added = 8346d. Therefore, £34 15s. 6d. = 8346d.

2. In £23 12s. 8d. 3 far., how many farthings?
3. In 18 lbs. 6 oz. 15 pwt. 14 grs., how many grains?
4. How many grains in 1 lb. 1 ℥ 2 ℈ 12 grains?
5. How many drams in 1 T. 3 qrs. 21 lbs. 10 drs.?
6. Reduce 14 cwt. 2 qrs. 20 lbs. to pounds.
7. Reduce 16 yards, 2 qrs. 2 nas. to nails.
8. In 12 E. Fr. 5 qrs. 1 na., how many nails?
9. In 8 E.E. 4 qrs. 3 nas., how many nails?
10. In 1 mile, how many feet?
11. In 1 mi. 5 fur. 35 rds. 5 yds. 2 ft. 6 in.; how many inches?
12. How many square poles in 102 acres, 3 R. 27 P.?
13. In 5 hhd. 20 gals. 3 qts. 1 pt.; how many pints?
14. In 2 pi. 2 gals. 2 gills; how many gills?
15. In 6 barrels, 25 gals. 3 qts. 1 pt. of beer; how many pints?
16. In 8 bushels, 2 pks. 5 qts. 1 pt.; how many pints?
17. In 2 weeks, 5 days, 5 hours, and 5 minutes; how many minutes?

18. In 1 day, how many minutes and seconds?
19. In 1 year, how many hours.
20 In 5 days, 4 hours, 45 seconds; how many seconds?
21. In 1 T. 1 lb. 1 dr.; how many drams?
22. In 1 acre; how many square feet?

Reduction Ascending.

1. In 647d., how many pounds, shillings and pence?

OPERATION.

```
    d.
12)647
------
 20)53  11d. rem.
------
     2  13s. 11d.
```

EXPLANATION.—There are 12 pence in 1s.; therefore, one-twelfth of the number of pence equals the number of shillings, which is 53s., and 11d. remaining. In 53s. how many pounds? There are 20s. in £1; therefore one-twentieth of the number of shillings equals the number of pounds, which is £2, and 13s. remaining. Therefore, 647d. = £2 13s. 11d.

2. In 16823 far., how many pounds, shillings, &c.?
3. In 84672 grs. Troy Weight; how many pounds, ounces &c.?
4. How many pounds, &c., Apothecaries' Weight, in 569ʒ?
5. In 1894763 dr. Avoidupois Weight; how many tons, cwt. &c.?
6. In 89643 lbs.; how many tons, cwt. &c.?
7. In 8467 nails; how many yards, qrs. &c.?
8. In 2706 nas.; how many E. E., qrs. &c.?
9. In 4762 nas.; how many E. Fr., qrs. &c.?
10. In 84672 feet; how many miles, &c.?
11. In 3647 rods; how many miles, furlongs, &c.?
12. In 1478 P.; how many acres, roods and poles?
13. In 165 qts.; how many gallons?
14. In 20042 gills; how many hogsheads, &c.?
15. In 17632 gallons; how many tons, &c.?
16. In 4007 pints of beer; how many barrels, &c.?
17. In 147 pints; how many bushels, &c.?
18. In 64 pints; how many bushels?
19. In 86400 seconds; how many days?
20. In 3146232 seconds; how many weeks, days, &c.?

CHAPTER IV.

PECULIAR PROPERTY OF THE NUMBER 9.

ART. **74.** *Any number is divisible by* 9, *when the sum of its digits is divisible by* 9. *Consequently, every number divided by* 9, *will give the same remainder as the sum of its digits divided by* 9.

Also, if from any number, the sum of its digits be subtracted, the remainder will be divisible by 9.

NOTE. The proof of the fundamental Rules of Arithmetic, is founded upon the above properties of the number 9, which we will now consider.

Take any number, as 765, which equals $700 + 60 + 5$.

$$\begin{aligned} \text{Now, } 700 &= 7\times100=7\times(99+1) &&= 7\times 99+7 \\ 60 &= 6\times 10=6\times(9+1) &&= 6\times 9+6 \\ 5 &= &&\ 5 \\ \hline \text{Hence, } 765 &= &&\ 7\times99+6\times9+7+6+5 \end{aligned}$$

But, $7 \times 99 + 6 \times 9$, which lacks the *sum* of the digits of the number, 765, of being equal to that number, is divisible by 9; since each of the expressions, 7×99 and 6×9, contains the factor 9. Hence, if the remaining part of the number, which is the *sum* of *its digits*, is divisible by 9 the number itself is divisible by 9. As every number can be separated into two parts,—the *sum of its digits*, and *another number*, divisible by 9, it follows that the same remainder will be found by dividing the number by 9, as is, by dividing its digits by 9: Also, if a number be diminished by the sum of its digits, the remainder will be divisible by 9.

MULTIPLICATION OF ABSTRACT POLYNOMIALS.

1. Multiply $5 + 7$ by $3 + 6$.

OPERATION.

$$\begin{array}{r} 5+7 \\ 3+6 \\ \hline 15+21+30+42 \text{ Product.} \end{array}$$

EXPLANATION.—Commence at the left, and multiply each term in the multiplicand successively, by each term in the multiplier. It is evident that the sum of these

several partial products, $(15 + 21 + 30 + 42 = 108,)$ is equal to the product of the sum of 5 and 7 by the sum of 3 and 6.

2. Multiply $2 + 3 + 4$ by $4 + 6 + 1$.
3. Multiply $8 + 6 + 2$ by $2 + 3 + 4$.
4. Multiply $4 + 6 + 7 + 8$ by $3 + 2 + 4$.
5. Multiply $1 + 2 + 3 + 4 + 5 + 6$ by $7 + 8 + 9$.
6. Multiply $9 + 8 + 7 + 6 + 5 + 4$ by $1 + 2 + 3 + 4$.

Miscellaneous Definitions.

ART. **75.** An INTEGER is any whole number.

ART. **76.** An EVEN *number* is any integer that contains 2 a whole number of times, without a remainder.

ART. **77.** An ODD *number* is any integer that does not contain 2 a whole number of times without a remainder. Hence, an odd number differs from an even number by a *unit*.

ART. **78.** A PRIME *number* is any integer that cannot be produced by multiplying two numbers together, each of which is greater than a unit; as 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, &c.

ART. **79.** A COMPOSITE *number* is an integer that can be produced by multiplying two numbers together, each of which is greater than a unit. Thus, 48 is a *composite*, number and may be produced either by multiplying together the *composite* factors, 6 and 8, or the *prime* factors 2, 2, 2, 2 and 3.

ART. **80.** The PRIME factors of a number are the *prime* numbers that are multiplied together to produce that number. The *prime* factors of 48 are 2, 2, 2, 2, and 3.

ART. **81.** All integers are *prime* numbers or *composite* numbers; and all composite numbers are composed of *prime factors*;—hence every integer is a *prime* number or composed of *prime factors*.

ART. **82.** A SQUARE *number* is a composite number, which is composed of two equal factors; as 9, $(=3\times3)$; 25, $(=5\times5)$; 49, $(=7\times7)$ &c.

ART. **83.** A CUBE *number* is a composite number, that is composed of three equal factors; as 8, $(=2\times2\times2)$; 27 $(=3\times3\times3)$ &c.

ART. **84.** The symbol $\therefore$, is equivalent to the word *therefore* or *consequently.*

PRIME NUMBERS.

ART. **85.** All prime numbers, except the digit 2, are odd numbers; consequently, they terminate with an odd digit; as, 1, 3, 5, 7, or 9. All numbers that end in 5 are divisible by 5, since the remainder, (if any next preceding the 5,) is a certain number of times 10, which added to 5, gives a number divisible by 5, since each of the numbers composing it, contains the factor 5;—therefore, *all prime numbers, except* 2 *and* 5 *must terminate* with 1, 3, 7, or 9.

Hence, to determine whether a given number is a prime, first, inspect its terminating figure, and if it differs from 1, 3, 7, or 9, it is a composite number; if not, it may still be *conposite;*—now, if we can find no number between 2, and another prime, the square of which is not less than the given number, that will divide it, the number is a prime.

ART. **86.** An odd number divided by an even number gives an odd number for a remainder; hence, if any *prime* number, except 2 and 3, be divided by 6 the remainder will be 1, 3, or 5; but, the remainder cannot be 3, as the number would then have been divisible by 3, since the divisor and remainder are each divisible by 3. Therefore, *any prime number, except* 2 *and* 3, *when divided by* 6 *will give* 1 *or* 5 *for a remainder.*

The following table is sufficiently extended for ordinary calculations.

TABLE OF PRIME NUMBERS.

1	163	383	619	877	1129	1433	1697	1999	2293	2609
2	167	389	631	881	1151	1439	1699	2003	2297	2617
3	173	397	641	883	1153	1447	1709	2011	2309	2621
5	179	491	643	887	1163	1451	1721	2017	2311	2633
7	181	409	647	907	1171	1453	1723	2027	2333	2647
11	191	419	653	911	1181	1459	1733	2029	2339	2657
13	193	421	659	919	1187	1471	1741	2039	2341	2659
17	197	431	661	929	1193	1481	1747	2053	2347	2663
19	199	433	673	937	1201	1483	1753	2063	2351	2671
23	211	439	677	941	1213	1487	1759	2069	2357	2677
29	223	443	683	947	1217	1489	1777	2081	2371	2683
31	227	449	691	953	1223	1493	1783	2083	2377	2687
37	229	457	701	967	1229	1499	1787	2087	2381	2689
41	233	461	709	971	1231	1511	1789	2089	2383	2693
43	239	463	719	977	1237	1523	1801	2099	2389	2699
47	241	467	727	983	1249	1531	1811	2111	2393	2707
53	251	479	733	991	1259	1543	1823	2113	2399	2711
59	257	487	739	997	1277	1549	1831	2129	2411	2713
61	263	491	743	1009	1279	1553	1847	2131	2417	2719
67	269	499	751	1013	1283	1559	1861	2137	2423	2729
71	271	503	757	1019	1289	1567	1867	2141	2437	2731
73	277	509	761	1021	1291	1571	1871	2143	2441	2741
79	281	521	769	1031	1297	1579	1873	2153	2447	2749
83	283	523	773	1033	1301	1583	1877	2161	2459	2753
89	293	541	787	1039	1303	1597	1879	2179	2467	2767
97	307	547	797	1049	1307	1601	1889	2203	2473	2777
101	311	557	809	1051	1319	1607	1901	2207	2477	2789
103	313	563	811	1061	1321	1609	1907	2213	2503	2791
107	317	569	821	1063	1327	1613	1913	2221	2521	2797
109	331	571	823	1069	1361	1619	1931	2237	2531	2801
113	337	577	827	1087	1367	1621	1933	2239	2539	2803
127	347	587	829	1091	1373	1627	1949	2243	2543	2819
131	359	593	839	1093	1381	1637	1951	2251	2549	2833
137	353	599	853	1097	1399	1657	1973	2267	2551	2837
139	359	601	857	1103	1409	1663	1979	2269	2557	2843
149	367	607	859	1109	1423	1667	1987	2273	2579	2851
151	373	613	863	1117	1427	1669	1993	2281	2591	2857
157	379	617	871	1123	1429	1693	1997	2287	2593	2861

Resolution of Composite Numbers into their Prime Factors.

1. What are the prime factors of 144?

OPERATION.

$$\begin{array}{r} 2)\underline{144} \\ 2)\underline{72} \\ 2)\underline{36} \\ 2)\underline{18} \\ 3)\underline{9} \\ 3 \end{array}$$

Explanation.—Divide the 144 by any prime number, greater than a unit, that is contained in it without a remainder; and divide this quotient in the same manner, and so continue dividing until the quotient obtained is a prime number. Then, a *unit*, the several *divisors*, and the last *quotient* will be the *prime factors* required. Proceeding, thus, we find the prime factors of 144 to be 1, 2, 2, 2, 2, 3, and 3.

2. What are the prime factors of 96?
3. What are the prime factors of 360?
4. What are the prime factors of 36?
5. What are the prime factors of 56?
6. What are the prime factors of 480?
7. What are the prime factors of 500?
8. What are the prime factors of 840?
9. Resolve 460 into its prime factors?
10. Resolve 680 into its prime factors?

Divisors or Measures of Numbers.

Art. **87.** A *divisor* or *measure* of any number is a number that is contained in it an exact number of times, without a remainder.

1. What are the divisors of 72?

Explanation.—We first find the *prime factors* of 72, which are 1, 2, 2, 2, 3, and 3. A number is evidently, divisible by its *prime factors* and the *products* arising from every combination of them. A unit and the factor 2 with all the products arising from 2, 2, and 2, gives 1, 2, 4, and 8. A unit and the factor 3 with all the products arising from 3 and 3, gives 1, 3,

and 9. The various products arising from the products already obtained, may be found by multiplication. Thus,

$$\begin{array}{l} 1+2+4+8 \\ 1+3+9 \\ \hline 1+2+4+8+3+6+12+24+9+18+36+72. \end{array}$$

Therefore, the divisors of 72, are 1, 2, 4, 8, 3, 6, 12, 24, 9, 18, 36, and 72.

2. What are the divisors of 48?
3. Find all the divisors of 96.
4. Find all the divisors of 144.
5. Find all the divisors of 360.

Common Measure or Divisor.

ART. **88.** A *Common measure* or *divisor* of two or more numbers, is any number that is contained in each of them a whole number of times without a remainder. Thus, 5 is a common measure of 10 and 15.

1. Find all the common measures, or divisors of 144 and 360.

OPERATION.

*2)	144	360
*2)	72	180
*2)	36	90
*3)	18	45
*3)	6	15
	2	5

EXPLANATION.—I first find the prime factors common to both numbers, which I have marked by *.—Since, 1, 2, 2, 2, 3, and 3 are the only prime factors that are common to 144 and 360, it follows that each of these factors, together with the products arising from their various combinations will be all the divisors of the two numbers, 1, 2, 4, and 8 are all the divisors arising from the common factors, 2, 2, and 2, 1, 3, and 9 are all the divisors arising from the common factors 3 and 3. The divisors arising from the combinations of the above divisors are found by multiplication. Thus,

$$\begin{array}{l} 1+2+4+8 \\ 1+3+9 \\ \hline 1+2+4+8+3+6+12+24+9+18+36+72 \end{array}$$

Hence; 1, 2, 4, 8, 3, 6, 12, 24, 9, 18, 36, and 72 are all the common divisors of 144 and 360.

2. Find all the common divisors of 24 and 48.
3. Find all the common divisors of 48, 96, and 120.
4. Find all the common divisors of 180, 360, and 480.
5. Find all the common divisors of 60, 120, and 180.

Greatest Common Measure.

ART. **89.** The GREATEST COMMON *measure*, or the GREATEST *common divisor* of two or more numbers, is the greatest number that is contained in each of them a whole number of times without a remainder. Thus, 7 is the greatest common measure of 35 and 42.

1. What is the greatest common measure of 126, 294, and 462?

OPERATION.

$$\text{The prime factors of} \begin{cases} 126=2^*\times3^*\times3\ \times7^* \\ 294=2^*\times3^*\times7^*\times7 \\ 462=2^*\times3^*\times7^*\times11 \end{cases}$$

EXPLANATION.—Since a number is divisible only by its prime factors and the various products of them, it follows that the product of all the factors that are common to any two or more numbers, must be the greatest common measure of these numbers.

The factors marked (*) are common to all these numbers, hence their product is the greatest common measure, or divisor of these numbers; which is $2\times3\times7=42$.

2. What is the greatest common measure of 462 and 770?

3. What is the greatest common divisor of 140, 105, and 245?

4. What is the greatest common divisor of 210, 350, and 770?

5. What is the greatest common measure of 286, 429, and 715?

ART. **90.** The greatest common measure of two or more numbers may also be found by

Dividing the larger number by the smaller, and the preceding divisor by the remainder, (if there be any,) and so continue

to divide the preceding divisor by the last remainder until nothing remains, then will the last divisor be the greatest common measure.

NOTE.—If there are more than two numbers; first find the greatest common measure of two of them, and then take this divisor and the remaining number and proceed as before.

6. What is the greatest common measure of 105 and 490?

OPERATION.

```
105)490(4
    420
    ---
     70)105(1
         70
        ---
         35)70(2
            70
           ---
             0
```

EXPLANATION.—If the remainder, (if any after division) will divide the preceding divisor, it will also divide the dividend, as that is the sum of a certain number of times the divisor and this remainder; it is also the greatest divisor of the two numbers, as that divisor is the same as the greatest divisor of the remainder and the preceding divisor.

This is rendered plain by inspection. Since 35 is contained in 70, it is contained in 105, (the sum of 70 and 35,) also in 490, (the sum of 70 and 4 times 105;) and is the greatest divisor of 105 and 490, as it is the greatest divisor of itself and 70 the largest numbers, taken at pleasure, that will produce the 105 and the 490. The $105=35+70$ and $490=4\times35+70$.

7. What is the greatest common measure of 3094 and 4420?

8. What is the greatest common measure of 296 and 407?

9. What is the greatest common measure of 360 and 480?

10. What is the greatest common measure of 268 and 286?

PRACTICAL QUESTIONS IN COMMON MEASURE.

1. A farmer has 120 bushels of wheat and 460 bushels of rye, which he is desirous of putting into boxes of equal size, without mixing the two kinds of grain. How much will the largest boxes that can be used hold?

2. A had $480; B $960; and C $360, which they were desirous of separating into different parcels, each contain-

ing the same number of dollars. What is the greatest number of dollars that each parcel can contain?

3. A speculator has in one place 240 acres of land, in another 480, and in another 640, and wishes to divide the whole into fields that shall be of equal size, and containing the greatest number of acres circumstances will allow. What will be the number of acres in each field?

Multiples.

Art. **91.** A Multiple of any number, is a number that will contain it a whole number of times, without a remainder. Thus, 21 is a multiple of 3.

Art. **92.** A common *multiple* of any two or more numbers, is a number that will, when divided by each of them, give an integer for a quotient. Thus, 24 is a common multiple of 4 and 8.

Art. **93.** The least common *multiple* of any two or more numbers, is the smallest number that will, when divided by each of them give an integer for a quotient. Thus 24 is the *least* common multiple of 6, 8, and 12.

Art. **94.** To find the least common multiple.

Place the numbers in a horizontal line. Then divide by any prime number greater than a unit that will divide the most of the given numbers without a remainder, and place the quotients thus obtained, with the undivided numbers in a line beneath; thus continue to divide until no number greater than a unit will divide any two or more of them without a remainder. Then the product of all the divisors, the last quotients, and the undivided numbers will be the least common multiple.

1. What is the least common multiple of 6, 9, and 30?

OPERATION.

$$\begin{array}{r|rrr} 2) & 6 & 9 & 30 \\ \hline 3) & 3 & 9 & 15 \\ \hline & 1 & 3 & 5 \end{array}$$

$2 \times 3 \times 3 \times 5 = 90$ is the least common multiple.

EXPLANATION.—Since the numbers 6, 9, and 30 are composed of the prime factors 2, 3, 3, and 5, or a certain number of them, it follows that their product will be a common multiple of these numbers;—and as all these factors are necessary to produce the above numbers, their product must be their *least common multiple.*

2. What is the least common multiple of 12, 16, and 20 ?
3. What is the least common multiple of 15, 30, and 9 ?
4. What is the least common multiple of 4, 8, 12, 16, and 20 ?
5. What is the least common multiple of 24, 48, 12 ?
6. What is the least common multiple of 18, 54, 27, and 12 ?
7. What is the least common multiple of 12, 90, and 45 ?
8. What is the least common multiple of 25, 45, 90 and 5 ?
9. What is the least common multiple of 64, 8, 81, 24, and 12 ?
10. What is the least common multiple of 60, 120, 48, 36, 24, 12, 6, and 4 ?

PRACTICAL QUESTIONS IN COMMON MULTIPLE.

1. What is the smallest sum of money for which I could purchase a number of hogs, at $9 each; a number of cows, at $27 each; or a number of horses, at $60 each;—and how many of each could I purchase for that sum ?
2. What is the smallest number of bushels of corn that will fill a number of barrels, each containing 3 bushels; a number of sacks, each containing 6 bushels; or a number of boxes, each containing 25 bushels ?
3. If one team can haul 12 barrels of sugar, at a load; another 15; and another 20;—what is the smallest number of barrels that will make a number of full loads for any of three teams ?

ABBREVIATED OPERATIONS IN ARITHMETICAL CALCULATIONS.

There are many abbreviated methods of calculation, in

particular cases, which will be of interest to the student, and of much importance to business men. We will mention a few of them to awaken a habit of observation on the part of the learner, that he may be enabled to discover others as circumstances may require.

ART. **95.** To multiply by 13, 14, &c., to 19.

Write the product of the *unit's* figure and the *multiplicand*, under the multiplicand, *one* place to the right and then add them.

Multiply 36472 by 16.

OPERATION.

```
36472 × 16
 218832
 ------
 583552
```

ART. **96.** If the multiplier is a unit followed by one or more ciphers and a *significant* figure, the multiplication can be performed by writing the product of the units' figure and the multiplicand as many places to the right of the multiplicand as there are intervening ciphers + 1.

Multiply 364273 by 104.

OPERATION.

```
364273   × 104
  1457092
 --------
 37884392 Ans.
```

Multiply 3468327 by 10007.

OPERATION.

```
3468327     × 10007
    24278289
 -----------
 34707548289 Ans.
```

ART. **97.** To multiply by 21, 31, 41, &c. to 91.

Place the product of the *ten's* figure and the multiplicand, under the multiplicand, so that its unit figure shall be under the *tens* of the multiplicand.

Multiply 3476 by 41.

OPERATION.

```
  3476  × 41
 13904
 ------
 142516
```

Should there be ciphers between the unit and the other significant figure of the multiplier; write the product of the significant figure *one* more place towards the left for every cipher.

Multiply 367432 by 701.

OPERATION.

```
   367432×701
 2572024
 ---------
 257569832 Ans.
```

Multiply 467321 by 80001.

OPERATION.

```
     467321 × 80001
 3738568
 -----------
 37386147321 Ans.
```

ART. **98.** To multiply by any number of 9's. From the multiplicand with as many ciphers annexed as there are 9's in the multiplier, subtract the multiplicand.

Multiply 34682 by 9999.

OPERATION.

```
 346820000
     34682
 ---------
 346785318 Ans.
```

EXPLANATION.—9999= 10000—1, consequently, by annexing four ciphers to the multiplicand, we have taken it *one* time more than we should have done, hence by subtracting the multiplicand gives the correct result.

ART. **99.** If the multiplier is an *Aliquot Part* of any number of *tens*, *hundreds*, or *thousands;* multiply by the number of *tens*, *hundreds*, or *thousands*, of which the multiplier is an aliquot *part*, then take the *same part* of the product thus found.

Aliquot Parts.

$12\frac{1}{2} = \frac{1}{8}$ of 100
$16\frac{2}{3} = \frac{1}{6}$ of 100
$25 = \frac{1}{4}$ of 100
$50 = \frac{1}{2}$ of 100
$75 = \frac{1}{4}$ of 300
$33\frac{1}{3} = \frac{1}{3}$ of 100
$13\frac{1}{3} = \frac{1}{3}$ of 40
&c. &c.

1. Multiply 3248 by $12\frac{1}{2}$.
$12\frac{1}{2} \times 8 = 100$, therefore $12\frac{1}{2}$ is *one-eighth* of 100.

OPERATION.

$$\begin{array}{r} 8)324800 \\ \hline 40600 \end{array} \text{ Ans.}$$

2. Multiply 86432 by 25.
$25 \times 4 = 100$, therefore 25 is *one-fourth* of 100.

OPERATION.

$$\begin{array}{r} 4)8643200 \\ \hline 2160800 \end{array} \text{ Ans.}$$

3. Multiply 846828 by $33\frac{1}{3}$.
$33\frac{1}{3} \times 3 = 100$, therefore $33\frac{1}{3}$ is *one-third* of 100.

OPERATION.

$$\begin{array}{r} 3)84682800 \\ \hline 28227600 \end{array} \text{ Ans.}$$

Art. **100.** Any number ending in 5, that is expressed by two figures, can be squared mentally.

The two right hand figures of the square number will always be 25, *the remaining figures on the left, will be the product of the digit in ten's place and a figure a unit greater.*

1. What is the square of 25.

$$25 \times 25 = 625$$

By inspecting the following multiplication, the reason of this method of squaring a number, expressed by two figures, that ends in 5, will become evident. This method of squaring a quantity will apply to a number expressed

by three, or more, figures; providing the figure occupying the unit's place is 5.

OPERATION.

$$
\begin{array}{r}
25 = \quad 20 + 5 \\
25 = \quad 20 + 5 \\
\hline
5\times20+25 \\
20\times20+\ 5\times20 \qquad \\
\hline
20\times20+10\times20+25
\end{array}
$$

REMARK.—Commence at the right and multiply by each figure separately.

The product is composed of $(10+20)$ times 20, $+$ 25, or 30 times 20 $+$ 25 $=$ 600 $+$ 25 $=$ 625.

45 squared = 2025
85 squared = 7225
&c. &c.
*125 squared = 15625.
&c. &c.

ART. **101.** The square of any NUMBER and a HALF is equal to the *product* of that *number* and a number a *unit* greater, increased by *one-fourth*.

$9\frac{1}{2}$ squared $= 9\times10+\frac{1}{4}=90\frac{1}{4}$
$8\frac{1}{2}$ " $= 8\times\ 9+\frac{1}{4}=72\frac{1}{4}$
&c. &c.

EXAMPLES IN ABBREVIATED MULTIPLICATION.

The pupil should be required to give the reason of all abbreviated operations.

1. Multiply 46234 by 13.
2. Multiply 8647 by 16.
3. Multiply 84672 by 19.
4. Multiply 46732 by 103.
5. Multiply 68472 by 107.
6. Multiply 723246 by 1009.
7. Multiply 67234 by 21.
8. Multiply 846232 by 41.
9. Multiply 8467231 by 81.
10. Multiply 102324 by 701
11. Multiply 347234 by 6001.
12. Multiply 4726846 by 80001.

* Consider the 12 on the left of the 5 as one number, and multiply it by a number a unit greater.

13. Multiply 4862321 by $12\frac{1}{2}$.
14. Multiply 846232 by $33\frac{1}{3}$.
15. Multiply 723246 by $16\frac{2}{3}$.
16. Multiply 8462342 by 25.
17. Square 25 mentally.
18. Square 35, 45, 55, 65, 75, 85, and 95, mentally.
19. Square 125, 135, 145, and 155, mentally.
20. Square $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{2}$, $10\frac{1}{2}$, $11\frac{1}{2}$, and $12\frac{1}{2}$, mentally.

ART. **102.** *Any* NUMBER *is* DIVISIBLE *by* ANOTHER *when it contains the same prime factors as that number*

Hence, to divide one number by another, resolve them into their prime factors and reject equal factors from each;—the product of the remaining factors of the dividend will be the quotient. Should the divisor not be a *measure* of the dividend, there will be factors remaining in the divisor also. In such cases, the product of the remaining factors in the dividend, divided by the product of the remaining factors in the divisor, will give the quotient.

1. Divide 1260 by 84.

OPERATION.

The prime factors of $\begin{cases} 1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7. \\ 84 \quad = 2 \times 2 \times 3 \times 7. \end{cases}$

$$\begin{array}{l} \text{Dividend,} \\ \text{Divisor,} \end{array} \frac{\cancel{2} \times \cancel{2} \times 3 \times \cancel{3} \times 5 \times \cancel{7}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{7}} = 15 \text{ quotient.}$$

2. Divide 3780 by 420.
3. Divide 6615 by 315.
4. Divide 46305 by 63.
5. Divide 2205 by 378.

OPERATION.

The prime factors of $\begin{cases} 2205 = 3 \times 3 \times 5 \times 7 \times 7. \\ 378 \quad = 2 \times 3 \times 3 \times 3 \times 7. \end{cases}$

$$\begin{array}{l} \text{Dividend,} \\ \text{Divisor,} \end{array} \frac{\cancel{3} \times \cancel{3} \times 5 \times \cancel{7} \times 7}{2 \times \cancel{3} \times \cancel{3} \times 3 \times \cancel{7}} = \frac{35}{6} = 5\tfrac{5}{6} \text{ Ans.}$$

REMARK.—The pupil, from what has been said, will readily discover other useful methods of abbreviating the operations of the Fundamenal rules.

PROPERTIES OF NUMBERS.

ART. **103.** *All numbers terminating on the right with* 0, 2, 4, 6, *or* 8, *are divisible by* 2; since each of the numbers 2, 4, 6, 8 and 10 contain a factor 2.

ART. **104.** *All numbers terminating on the right with* 0, *or* 5, *are divisable by* 5; since each of the numbers, 5 and 10, contain a factor 5.

ART. **105.** *If the two right-hand figures of any number are divisible by* 4, *the whole number will be divisible by* 4. For, if there is any remainder next preceding the two figures on the right, it will be a certain number of times 100; and 4 is a measure of 100, since the 100 contains the same prime factors as 4; and as it is also, a measure of the two right-hand figures; it is a measure of the whole number.

ART. **106.** *If the three right-hand figures of any number are divisible by* 8, *the whole number will be divisible by* 8. For the remainder, (if any,) next preceding the three figures on the right, will be a certain number of times 1000; and 8 is the measure of 1000, since the 1000 contains the same prime factors as 8; the 8 being also, a measure of the three right-hand figures;—it must be a measure of the whole number.

ART. **107.** *When the sum of the digits of any number is divisible by* 3 *or* 9, *the number itself is divisible by* 3 *or* 9. (For the reason, see Art. 74.)

CHAPTER V.

FRACTIONS.

ART. **108.** A FRACTION is an expression denoting one or more of the equal parts into which a unit, or any collection of units may be divided.

There are two kinds of fractions employed in Arithme-

tical calculations; namely, COMMON FRACTIONS and DECIMAL FRACTIONS.

The Common Fractions have generally been called *Vulgar Fractions;* the word *vulgar*, meaning *common*.

COMMON FRACTIONS.

ART. **109.** A COMMON FRACTION consists of two numbers, one written above the other, with a short horizontal line between them.

The number above the line is called the *Numerator*, and shows how many of these parts are considered, or taken.

The number below the line is called the *Denominator*, and shows into how many equal parts the unit or *integer* is divided.

$\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{5}{6}$, &c., are Common Fractions and are read

Numerator.	1	One	
Denominator.	3	Third of One.	
Numerator.	2	Two	or one fifth of 2.
Denominator.	5	Fifths of one,	
Numerator.	4	Four	or one seventh of 4.
Denominator.	7	Sevenths of one,	
Numerator.	5	Five	or one sixth of 5.
Denominator.	6	Sixths of one,	

By inspecting the above expressions, it will be observed that they are unperformed operations in division. The denominators being the divisors, and the numerators the dividends. Hence, a Common Fraction may be considered a method of expressing division. (See Remark 2, Art. 45.)

In the fraction $\frac{5}{9}$, the numerator, 5, is the dividend, and the denominator, 9, is the divisor

ART. **110.** When the numerator of a fraction is less than the denominator, the value is less than a unit; as, $\frac{3}{5}$.

2. When the numerator of a fraction is equal to the denominator, the value is a unit; as, $\frac{3}{3} = 1$.

3. When the numerator of a fraction is greater than

the denominator, the value is greater than a unit; as, $\frac{7}{5}=1\frac{2}{5}$.

ART. **111.** There are FIVE kinds of Common Fractions, namely; *Proper*, *Improper*, *Simple*, *Compound*, and *Complex.*

A PROPER FRACTION is one, the numerator of which is less than the denominator; therefore, its value is less than a unit; as, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, &c.

An IMPROPER FRACTION is a fraction, the numerator of which is equal to, or greater than the denominator; therefore, its value is equal to, or greater than a unit; as, $\frac{7}{3}$, $\frac{5}{4}$, $\frac{7}{7}$, &c.

A SIMPLE FRACTION is one in which the numerator and denominator each consist of an integer; and may be either a proper or an improper fraction.

A COMPOUND FRACTION is a fraction of a fraction, or any number of fractions connected by the word of; as, $\frac{3}{4}$ of $\frac{2}{5}$ of $\frac{7}{8}$ of $\frac{8}{9}$.

A COMPLEX FRACTION is a fraction that has a fraction, or a mixed number, in the numerator or denominator, or in both; as $\dfrac{\frac{3}{4}}{4\frac{1}{2}}$; $\dfrac{2}{3\frac{1}{3}}$ &c.

A MIXED NUMBER consists of an integer and a fraction; as, $7\frac{1}{2}$, $24\frac{2}{5}$, &c.

ART. **112.** The *Terms* of a fraction are two in number, the numerator and the denominator.

To invert a fraction, cause the numerator and the denominator to change places. Thus, $\frac{3}{5}$ when inverted, becomes $\frac{5}{3}$.

Any whole number may be expressed fractionally by writing a unit below it for a denominator. Thus,

$4=$	$\frac{4}{1}$,	and is read	4 ones, or four.
$7=$	$\frac{7}{1}$,	" " "	7 ones, or seven.
$9=$	$\frac{9}{1}$,	" " "	9 ones, or nine.
$12=$	$\frac{12}{1}$,	" " "	12 ones, or twelve.

Reduction of Common Fractions.

Art. **113.** Reduction of Fractions is changing them from one form to another while their value remains the same.

Art. **114.** Reduction of Mixed Numbers to Improper Fractions.

1. In $25\frac{2}{3}$ how many thirds?

Solution.—In 1 there are $\frac{3}{3}$, and in 25 there are 25 times $\frac{3}{3}=\frac{75}{3}$, which added to $\frac{2}{3}=\frac{77}{3}$, consequently, $25\frac{2}{3}=\frac{77}{3}$.

2. In $43\frac{3}{4}$, how many fourth?
3. In $146\frac{3}{7}$, how many sevenths?
4. In $236\frac{1}{2}$, how many halves?
5. Reduce $684\frac{2}{9}$ to an improper fraction.
6. Reduce $783\frac{7}{8}$ to an improper fraction.
7. Reduce $1862\frac{4}{13}$ to an improper fraction.
8. Reduce $2864\frac{5}{17}$ to an improper fraction.
9. Reduce $8623\frac{13}{264}$ to an improper fraction.
10. Reduce $76432\frac{17}{168}$ to an improper fraction.

Art. **115.** Reduction of Improper Fractions to Mixed Numbers.

1. Reduce $\frac{247}{3}$ to a mixed number.

Solution.—In one there are $\frac{3}{3}$; therefore, 1 third of the *number* of *thirds*, equals the number of whole ones. 1 third of $247=82\frac{1}{3}$. Hence, $\frac{247}{3}=82\frac{1}{3}$.

2. Reduce $\frac{867}{3}$ to a mixed number.
3. Reduce $\frac{847}{4}$ to a mixed number.
4. Reduce $\frac{767}{4}$ to a mixed number.
5. Reduce $\frac{847}{5}$ to a mixed number.
6. Reduce $\frac{8176}{6}$ to a mixed number.
7. Reduce $\frac{1472}{9}$ to a mixed number.
8. Reduce $\frac{1846}{11}$ to a mixed number.
9. Reduce $\frac{17846}{15}$ to a mixed number.
10. Reduce $\frac{276847}{35}$ to a mixed number.

PROPOSITIONS.

ART. **116.** PROPOSITION 1.—*Multiplying the numerator of a fraction by any number, the denominator remaining the same, multiplies the value of the fraction by that number.*

The denominator of a fraction shows into how many equal parts the quantity is divided, and therefore, designates the size of the parts compared with that quantity. The numerator shows how many of these parts are taken; hence, multiplying the numerator increases the value of the fraction as many times as there are units in the multiplier, if the *denominator*, that is, the size of the parts remains the same.

PROPOSITION 2.—*Dividing the denominator of a fraction by any number, the numerator remaining the same, multiplies the value of the fraction by that number.*

The numerator of a fraction shows how many parts are taken, and the denominator measures the size of these parts, compared with the quantity referred to; if we divide the denominator by any number it diminishes the number of parts into which the thing is divided, and, therefore, increases their size proportionally; hence, the value of the fraction is multiplied by the same number, if the *numerator*, that is, the number of parts taken, remains the same.

PROPOSITION 3.—*Multiplying the denominator of a fraction by any number, the numerator remaining the same, divides the value of the fraction by that number.*

The numerator of a fraction shows how many parts are taken, and the denominator shows into many equal parts the unit or thing is divided, and therefore, designates the size of these parts compared with the unit or thing divided. Multiplying the denominator by any number, increases the number of parts into which the thing is divided, as many times as there are units in the multiplier, and necessarily diminishes their size proportionally; therefore, the value of the fraction is divided, if the numerator remains unchanged.

PROPOSITION 4.—*Dividing the numerator of a fraction by any number, the denominator remaining the same, divides the value of the fraction by that number.*

Since, the denominator shows into how many equal parts the unit or thing is divided, and the numerator shows how many of these parts are taken; it follows, that dividing the numerator,

divides the value of the fraction, as it diminishes the number of parts taken while their size remains the same.

REMARK.—By inspecting proposition 1 and 3, we deduce

PROPOSITION 5.—*Multiplying the numerator and denominator of any fraction by the same number, does not change the value of the fraction.*

REMARK.—By. inspecting proposition 2 and 4, we deduce

PROPOSITION 6.—*Dividing both numerator and denominator of any fraction by the same number, does not change the value of the fraction.*

MULTIPLICATION OF FRACTIONS BY INTEGERS.

ART. **117.** According to propositions 1st and 2d, to *to multiply* a fraction by any integer; *Multiply the numerator of the fraction by that number,—or divide its denominator by the same number.*

1. If 1 bushel of apples cost $\$\frac{4}{7}$, what will 15 bushels cost?

SOLUTION.—If 1 bushel cost $\$\frac{4}{7}$, 15 bushels will cost $15 \times \$\frac{4}{7} = \frac{60}{7} = \$8\frac{4}{7}$.

2. If 1 barrel of sugar cost $\$12\frac{4}{9}$, (equal to $\$\frac{112}{9}$,) what will 3 barrels cost?

SOLUTION.—If 1 barrel cost $\$\frac{112}{9}$, 3 barrels will cost 3 times $\$\frac{112}{9} = \frac{112}{3} = \$37\frac{1}{3}$.

3. What cost 25 bushels of peaches, at $\$\frac{3}{4}$ a bushel?
4. What cost 18 yards of broadcloth, at $\$6\frac{2}{8}$ a yard?
5. What cost 47 barrels of flour, at $\$5\frac{5}{6}$ a barrel?
6. What cost 15 cows, at $25\frac{3}{5}$ each?
7. What cost 52 barrels of cider, at $\$7\frac{3}{4}$ a barrel?
8. What cost 17 hogsheads of molasses, at $\$47\frac{8}{9}$ a hogshead?

DIVISION OF FRACTIONS BY INTEGERS.

ART. **118.** According to Propositions 3d and 4th, *to divide a fraction by any integer; Divide the numerator of the fraction by that number,—or muitiply the denominator by the same number.*

1. If 16 yards of cloth cost $\$37\frac{1}{3}$, what will 1 yard cost?

2. If 13 yards of ribbon cost $32\frac{1}{2}$ cents, how much is that a yard?

3. If 8 books cost $\$13\frac{2}{3}$, how much is that a piece?

4. If 14 lbs. of sugar cost $93\frac{3}{4}$ cents, how much is that a pound?

5. If 17 barrels of sugar cost $\$282\frac{3}{4}$, how much is that a barrel?

6. What cost 1 horse, if 19 horses cost $\$7824\frac{3}{4}$?

7. What cost 5 oranges, if 15 cost $35\frac{1}{2}$ cents?

8. What cost 6 acres of land if 17 acres cost $\$403\frac{3}{4}$?

9. What cost 3 bushels of flax-seed, if 7 bushels cost $\$27\frac{2}{3}$?

10. What cost 4 horses if 12 cost $\$2104\frac{1}{2}$?

ART. **119.** According to proposition 6th, to reduce a fraction to its lowest terms:

Divide both numerator and denominator by any number greater than a unit, that is contained in them both without a remainder; proceed in the same way with the successive results, until the operation can be carried no farther. (See Articles 103 to 107, Properties of numbers.)

Or,

Find the greatest common measure of the numerator and denominator (by Art. 89 or Art. 90) *and divide them by it.*

Or,

Resolve both numerator and denominator into their prime factors and reject equal factors from each; (See Art. 102) the result will be the fraction reduced to its lowest terms.

1. Reduce $\frac{360}{600}$ to its lowest terms.

OPERATION by the last method.

The prime factors of $\left\{\frac{360=\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{5}\times\cancel{3}\times3}{600=\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{5}\times5\times\cancel{3}}\right.=\frac{3}{5}$ Ans

2. Reduce $\frac{336}{384}$ to its lowest terms.

3. Reduce $\frac{120}{180}$ to its lowest terms.

4. Reduce $\frac{64}{72}$ to its lowest terms.

5. Reduce $\frac{288}{360}$ to its lowest terms.

6. Reduce $\frac{216}{252}$ to its lowest terms.

7. Reduce $\frac{960}{1536}$ to its lowest terms.
8. Reduce $\frac{352}{384}$ to its lowest terms.
9. Reduce $\frac{3172}{2196}$ to its lowest terms.
10. Reduce $\frac{10212}{10656}$ to its lowest terms.

ART. **120.** REDUCTON OF COMPOUND FRACTIONS TO SIMPLE ONES.

REMARK.—The word *of* in the following questions, is equivalent to the sign of multiplication; therefore, in its stead the sign $\times$, may be used.

1. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ to a simple fraction.

OPERATION.

$$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15} \text{ Ans}$$

SOLUTION.—$\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, and $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times $\frac{1}{15}$, which is $\frac{4}{15}$; and if $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}$ is twice $\frac{4}{15}$, which are $\frac{8}{15}$. Therefore, $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.

REMARK.—From the above solution we observe, that to reduce a compound fraction to a simple one, *we multiply all the numerators together for a new numerator, and all the denominators for a new denominator.*

2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction.
3. Reduce $\frac{7}{8}$ of $\frac{7}{14}$ to a simple fraction.
4. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{9}{10}$ to a simple fraction.
5. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{7}{8}$ to a simple fraction.
6. Reduce $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

CANCELLATION.

ART. **121.** TO REDUCE A COMPOUND FRACTION TO A SIMPLE ONE BY CANCELLATION.

Write the fraction down with the sign of multiplication between them, and cancel or reject all the factors that are common to the numerators and denominators, (which by Proposition 6*th, under Art.* 116, *does not change the value of the fraction;) then multiply the remaining numerators together for a new numerator, and the remaining denominators for a new denominator.*

Take for illustration the 6th example.

$$\frac{1}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{5}=\frac{1}{5}$$

EXPLANATION.—First cancel the 3 of the numerator against the 3 of the denominator, by drawing a line across them ; then cancel the 4 of the numerator against the 4 of the denominator in the same manner. As there are no more factors common to both numerator and denominator,—multiply the remaining numerators together for a new numerator, and the remaining denominators, for a new denominator.

ART. **122.** *If any numerator and denominator have a common divisor, divide them both by this divisor, and use the quotients as a new fraction.*

6. Reduce $\frac{5}{7}$ of $\frac{7}{10}$ of $\frac{12}{16}$ to a simple fraction.

OPERATION.

$$\frac{\overset{*1}{\cancel{5}}}{\cancel{7}}\times\frac{\cancel{7}}{\underset{3}{\cancel{15}}}\times\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}=\frac{2}{9}$$

EXPLANATION.—5 being a divisor of the 5 in the numerator and the 15 in the denominator, we divide them both by 5 and cancel the 5 and the 15, and consider the quotients, 1 and 3, arising from this division, instead of the 5 and 15. Next cancel the 7 in the numerator and the 7 in the denominator. We observe that 4 is a common measure of the 8 in the numerator, and of the 12 in the denominator ; therefore, we divide by it, and cancel the 8 and 12, and place the quotients in their proper places. As there are no more factors common to both numerator and denominator, nor any number that will divide them both without a remainder, we multiply all the remaining numerators together for a new numerator, and all the remaining denominators for a new denominator, and obtain for the answer $\frac{2}{9}$.

7. Reduce $\frac{3}{4}$ of $\frac{7}{3}$ of $\frac{6}{14}$ of $\frac{8}{12}$ to its simplest form.

8. Reduce $\frac{4}{7}$ of $\frac{8}{9}$ of $\frac{12}{16}$ of $\frac{3}{4}$ to its simplest form.

9. Reduce $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{12}{17}$ of $\frac{3}{4}$ of $\frac{17}{4}$ of $\frac{9}{27}$ to its simplest form.

10. Reduce $\frac{3}{5}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{12}$ of $\frac{8}{9}$ of $\frac{24}{18}$ to its simplest form.

11. What is the value of the compound fraction, $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{6}$ of $\frac{7}{12}$ of $\frac{24}{10}$ of $\frac{9}{27}$?

* In practice we do not write the quotient when it is a unit.

12. What is the value of the compound fraction $\frac{5}{7}$ of $\frac{25}{6}$ of $\frac{12}{30}$ of $\frac{15}{45}$ of $\frac{35}{53}$ of $\frac{15}{25}$?

REMARK.—All whole and mixed numoers that occur in compound fractions, must be changed to improper fractions before the required reduction is performed.

13. Reduce $1\frac{3}{11}$ of $\frac{3}{7}$ of $\frac{6}{12}$ of $2\frac{3}{4}$ to its simplest form.

14. Reduce $\frac{56}{168}$ of $3\frac{1}{3}$ of $1\frac{9}{15}$ of $7\frac{1}{2}$ to its simplest form.

15. Reduce $\frac{8}{13}$ of $4\frac{1}{2}$ of $3\frac{1}{4}$ of $\frac{5}{26}$ of $\frac{7}{9}$ of $1\frac{1}{8}$ of $3\frac{1}{3}$ to its simplest form.

16. Reduce $\frac{5}{6}$ of $9\frac{1}{2}$ of $\frac{4}{38}$ of $\frac{16}{20}$ of $\frac{15}{25}$ to its simplest form.

17. Reduce $\frac{9}{11}$ of $3\frac{3}{17}$ of $\frac{7}{18}$ of $\frac{6}{14}$ of $3\frac{1}{6}$ to its simplest form.

18. Reduce $\frac{9}{11}$ of $7\frac{1}{2}$ of $\frac{8}{27}$ of $12\frac{3}{4}$ of $\frac{4}{17}$ to its simplest form.

19. Reduce $\frac{96}{144}$ of $\frac{7}{32}$ of $\frac{8}{7}$ of $\frac{18}{58}$ of $9\frac{3}{4}$ to its simplest form.

A COMMON DENOMINATOR.

ART. **123.** Two or more fractions have a Common Denominator, when they have the *same number* for a denominator.

1. Reduce $\frac{3}{4}$ and $\frac{4}{5}$ to equivalent fractions having a common denominator.

OPERATION.

$$\frac{3}{4}, \frac{4}{5} = \frac{15}{20}, \frac{16}{20} \text{ or } = \frac{15, 16}{20}$$

EXPLANATION.—I first multiply the denominators together for a common denominator,—4 times 5 are 20, the common denominator. Since, I have multiplied the denominator 4, of the fraction $\frac{3}{4}$ by 5, to preserve the value of the fraction, I multiply the numerator 3, by the same number. 5 times 3 are 15; therefore, $\frac{3}{4}$ equals $\frac{15}{20}$. I have multiplied the denominator 5, of the fraction $\frac{4}{5}$ by 4, and to preserve the value of the fraction, I multiply the numerator 4, by the same number. 4 times 4 are 16; therefore, $\frac{4}{5}$ equals $\frac{16}{20}$.

From the above explanation, to reduce fractions to equivalent ones have a common denominator, we infer that we should *Multiply all the denominators together for a common denominator, and each numerator by all the denominators except its own, for a new numerator.*

REMARK.—It is readily observed that, by the above process, both numerator and denominator of each fraction is multiplied by the same number, which by Proposition 5, under Art. 116, does not change the value of the fraction.

2. Reduce $\frac{2}{3}$ and $\frac{3}{5}$ to equivalent fractions having a common denominator.

3. Reduce $\frac{4}{7}$ and $\frac{3}{4}$ to equivalent fractions having a common denominator.

4. Reduce $\frac{5}{6}$ and $\frac{5}{9}$ to equivalent fractions having a common denominator.

5. Reduce $\frac{6}{7}$ and $\frac{7}{9}$ to equivalent fractions having a common denominator.

6. Reduce $\frac{9}{13}$ and $\frac{9}{17}$ to equivalent fractions having a common denominator.

7. Reduce $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{3}{4}$ to equivalent fractions having a common denominator.

8. Reduce $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{8}$ to equivalent fractions having a common denominator.

9. Reduce $\frac{3}{5}$, $\frac{3}{7}$ and $\frac{3}{8}$ to equivalent fractions having a common denominator.

10. Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{5}{8}$ to equivalent fractions having a common denominator.

THE LEAST COMMON DENOMINATOR.

ART. **124.** The *least common denominator* of two or more fractions, is the least common *multiple* of their denominators. Hence, to find the least common denominators of two or more fractions, *reduce compound fractions to simple ones, whole and mixed numbers, to improper fractions, and all to their lowest terms; then find the least common multiple of the denominators of the fractions, (by Art. 94,) and it will be their least common denominator.*

1. Reduce $\frac{5}{12}$, $\frac{5}{8}$, and $\frac{7}{18}$, to equivalent fractions having the least denominator.

OPERATION.

$$2\underline{)\frac{5}{12},\ \frac{5}{6},\ \frac{7}{18}} = \frac{15}{36},\ \frac{30}{36},\ \frac{14}{36}, \text{ or write them thus, } \frac{15,\ 30,\ 14}{36}$$

$$3\underline{)6\quad 3\quad 9}$$

$$\ \ 2\quad 1\quad 3$$

$2 \times 3 \times 2 \times 3 = 36$, the least common denominator.

SOLUTION.—The remaining part of the work is to reduce each of the given fractions to THIRTY SIXTHS, without changing their value. This can be done by multiplying the terms of each fraction by a number that will cause its denominator to become, 36. (See Art. 116, Proposition 5.) To find what number I must multiply 12 by to produce 36, I divide the 36 by 12, and find it to be 3. Multiply both numerator and denominator of $\frac{5}{12}$ by 3, gives $\frac{15}{36} = \frac{5}{12}$. Proceed in the same way with the remaining fractions.

2. Reduce $\frac{5}{7}$, $\frac{9}{14}$ and $\frac{5}{28}$ to equivalent fractions having the least common denominator.

3. Reduee $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{5}{12}$ to equivalent fractions having the least common denominator.

4. Reduce $\frac{9}{13}$, $\frac{9}{20}$, and $\frac{9}{36}$ to equivalent fractions having the least common denominator.

5. Reduce $\frac{6}{7}$, $9\frac{8}{14}$, and $\frac{25}{28}$ to equivalent fractions having the least common denominator.

6. Reduce $\frac{7}{10}$, $2\frac{3}{5}$, and $3\frac{4}{15}$ to equivalent fractions having the least common denominator.

7. Reduce $2\frac{4}{5}$, $3\frac{2}{10}$, and $3\ \frac{6}{12}$ to equivalent fractions having the least common denominator.

8. Reduce $\frac{6}{8}$ of $\frac{2}{3}$, $\frac{7}{8}$ of $\frac{4}{14}$, and $\frac{2}{3}$ of $\frac{3}{4}$ to equivalent fractions having the least common denominator.

9. Reduce $\frac{1}{2}$ of $\frac{6}{7}$ of $\frac{7}{3}$, $\frac{5}{8}$ of $\frac{4}{5}$, $\frac{2}{3}$ of $\frac{6}{8}$, and $\frac{4}{5}$ of $\frac{5}{6}$ to equivalent fractions having the least common denominator.

10. Reduce $2\frac{4}{5}$ of $\frac{5}{8}$, $\frac{8}{15}$ of $\frac{5}{4}$, $3\frac{2}{3}$ of $\frac{7}{22}$ of $\frac{6}{7}$, and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to equivalent fractions having the least common denominator.

ADDITION OF COMMON FRACTIONS.

ART. **125.** *Addition of common fractions* is the process of finding the sum of two or more fractions.

1. What is the sum $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$ and $\frac{2}{4}$?

OPERATION.

$$\frac{1}{4} + \frac{3}{4} + \frac{5}{4} + \frac{2}{4} = \frac{11}{4}, \text{ or } 2\frac{3}{4}. \text{ Ans.}$$

2. What is the sum of $\frac{4}{6}$, $\frac{2}{6}$, $\frac{5}{6}$, $\frac{12}{6}$, and $\frac{8}{6}$?

3. What is the sum of $\frac{4}{7}$, $\frac{5}{7}$, $\frac{9}{7}$, $\frac{12}{7}$, and $\frac{9}{7}$?

4. What is the sum of $\frac{4}{17}$, $\frac{5}{17}$, $\frac{11}{17}$, $\frac{15}{17}$, $\frac{12}{17}$, and $\frac{14}{17}$?

5. What is the sum of $\frac{15}{18}$, $\frac{16}{18}$, $\frac{25}{18}$, $\frac{32}{18}$, $\frac{12}{18}$, $\frac{17}{18}$, and $\frac{19}{18}$?

REMARK.—Reduce *compound fractions* to *simple ones*, and *mixed numbers* to *improper fractions*, and *all* to their lowest terms. Also, reduce fractions that have different denominators to equivalent ones having the *least common denominator*.

6. What is the sum of $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{7}{12}$?

OPERATION.

$$2)\frac{3}{4} + \frac{7}{8} + \frac{7}{12} = \frac{18 + 21 + 14}{24} = \frac{53}{24} = 2\frac{5}{24}. \text{ Ans.}$$

$$\begin{array}{r|lll} 2 & 2, & 4, & 6 \\ \hline & 1, & 2, & 3 \end{array}$$

$2 \times 2 \times 2 \times 3 = 24$, the least common denominator.

7. What is the sum of $\frac{2}{9}$, $\frac{7}{8}$, and $\frac{5}{12}$?

8. What is the sum of $\frac{7}{8}$, $4\frac{3}{4}$, and $2\frac{1}{3}$?

9. What is the sum of $4\frac{1}{4}$, $8\frac{2}{3}$, and $3\frac{5}{6}$?

10. What is the sum of $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{7}{6}$, and $\frac{2}{3}$ of $\frac{3}{4}$?

11. What is the sum of $\frac{5}{6}$ of $\frac{3}{10}$, $\frac{7}{8}$ of $\frac{16}{7}$, $\frac{1}{2}$ of $\frac{4}{3}$, and $\frac{7}{8}$?

12. What is the sum of $3\frac{1}{3}$ of $5\frac{1}{4}$, $6\frac{5}{12}$ of $\frac{6}{7}$, and $2\frac{1}{6}$ of $\frac{6}{12}$?

13. What is the sum of $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{8}{9}$, $\frac{7}{8}$, and $\frac{5}{6}$?

14. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{7}$, $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$?

REMARK.—When two fractions are to be added, the numerator of each being a unit, it may be done mentally, by taking the sum of the denominators for a new numerator and their product for a denominator.

Thus, the sum of $\frac{1}{5}$ and $\frac{1}{7} = \frac{7 + 5}{7 \times 5} = \frac{12}{35}$

15. What is the sum of $\frac{1}{3}$ and $\frac{1}{5}$?

16. What is the sum of $\frac{1}{4}$ and $\frac{1}{6}$?

17. What is the sum of $\frac{1}{5}$ and $\frac{1}{7}$?

18. What is the sum of $\frac{1}{7}$ and $\frac{1}{8}$?

19. What is the sum of $\frac{1}{6}$ and $\frac{1}{7}$?

SUBTRACTION OF COMMON FRACTIONS.

ART. **126.** *Subtraction of common fractions* is the method of finding the difference between two fractions.

1. From $\frac{5}{9}$ subtract $3\frac{3}{9}$.

OPERATION.

$\frac{5}{9} - \frac{3}{9} = \frac{2}{9}$. Ans.

2. From $\frac{5}{6}$ take $\frac{2}{6}$.
3. From $\frac{5}{7}$ take $\frac{4}{7}$.
4. From $\frac{12}{17}$ take $\frac{9}{17}$.
5. From $\frac{14}{19}$ take $\frac{8}{19}$.

REMARK.—Reduce *compound fractions* to *simple ones*, and *mixed number* to *improper* fractions, and *all* to their lowest terms. Also, reduce fractions that have different denominations to equivalent ones having the *least common denominator*.

6. From $\frac{7}{8}$ take $\frac{5}{12}$.

OPERATION.

$$\begin{array}{r|rr} 2) & \frac{7}{8} & \frac{5}{12} \\ \hline 2) & 4 & 6 \\ \hline & 2 & 3 \end{array} = \frac{21-10}{24} = \frac{11}{24} \text{ Ans.}$$

$2 \times 2 \times 2 \times 3 \times = 24$, the least common denominator.

7. From $\frac{8}{9}$ take $\frac{3}{7}$.
8. From $\frac{7}{8}$ take $\frac{2}{3}$.
9. From $1\frac{1}{2}$ take $\frac{3}{4}$.
10. From $4\frac{1}{7}$ take $1\frac{2}{3}$.
11. From $8\frac{7}{8}$ take $6\frac{3}{4}$.
12. From $7\frac{4}{5}$ take $6\frac{7}{9}$.
13. From $\frac{2}{3}$ of $\frac{3}{4}$ take $\frac{1}{6}$ of $\frac{3}{4}$.
14. From $\frac{3}{5}$ of $\frac{5}{12}$ take $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{1}{8}$.
15. From $\frac{2}{3}$ of $1\frac{1}{6}$ of $4\ \frac{7}{8}$ of $\frac{64}{49}$ of $\frac{1}{10}$.
16. From $9\frac{1}{10}$ of $4\frac{1}{5}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of $4\frac{1}{2}$.

REMARK.—When both the fractions have a *unit* for their numerator, the subtraction may be performed mentally by placing the product of the denominators under their difference.

Thus, $\frac{1}{5} - \frac{1}{8} = \frac{8-5}{40} = \frac{3}{40}$.

17. From $\frac{1}{2}$ take $\frac{1}{3}$.
18. From $\frac{1}{3}$ take $\frac{1}{5}$.
19. From $\frac{1}{8}$ take $\frac{1}{14}$.
20. From $\frac{1}{9}$ take $\frac{1}{16}$.
21. From $\frac{1}{7}$ take $\frac{1}{12}$.

22. From $\frac{1}{14}$ take $\frac{1}{43}$.
23. From $\frac{1}{16}$ take $\frac{1}{56}$.

Multiplication of Common Fractions.

ART. **127.** *Multiplication of common fractions* is the method of finding the product of two or more fractions, or of integers and fractions.

ART. **128.** To multiply one fraction by another, or an integer by a fraction. First:

Reduce compound fractions to simple ones, and whole or mixed numbers to improper fractions. Then proceed as in the reduction of compound fractions. (See Art. 121.)

1. Multiply $\frac{4}{3}$, $\frac{5}{7}$, $\frac{12}{16}$, $\frac{12}{18}$, and $\frac{35}{25}$, together.

OPERATION BY CANCELLATION.

$$\frac{\cancel{4}}{3}\times\frac{\cancel{5}}{\cancel{7}}\times\frac{\overset{2}{\cancel{12}}}{\underset{\cancel{4}}{\cancel{16}}}\times\frac{\overset{\cancel{3}}{\cancel{12}}}{\underset{\cancel{3}}{\cancel{18}}}\times\frac{\overset{\cancel{5}}{\cancel{35}}}{\underset{\cancel{5}}{\cancel{25}}}=\frac{2}{3}\text{ Ans.}$$

2. Multiply $\frac{2}{3}$ by $\frac{6}{8}$.
3. Multiply $\frac{6}{8}$ by $\frac{16}{18}$.
4. Multiply $\frac{4}{5}$ by $\frac{10}{16}$.
5. Multiply $3\frac{1}{2}$ by $4\frac{1}{3}$.
6. Multiply together $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$.
7. Multiply together $\frac{3}{4}$, $\frac{7}{8}$, $\frac{2}{3}$, $\frac{16}{14}$, and $\frac{3}{5}$.
8. Multiply together $2\frac{1}{2}$, $\frac{14}{16}$, $8\frac{1}{3}$ and $\frac{15}{17}$.
9. Multiply together $4\frac{1}{8}$, $\frac{4}{11}$, $5\frac{1}{2}$, $8\frac{1}{3}$, and $\frac{7}{25}$.
10. Multiply together $\frac{3}{4}$, $9\frac{1}{3}$, $7\frac{3}{5}$, $\frac{8}{14}$, $\frac{10}{19}$, and $7\frac{1}{2}$.

PRACTICAL QUESTIONS IN MULTIPLICATION OF FRACTIONS.

REMARK.—In business transactions it is customary to add 1 cent when the fraction is equal to or greater than a half of a cent, and to omit it when it is less than the half of a cent.

1. What cost 42 bushels of apples, at $63\frac{2}{7}$ cents a bushel?
2. What cost $7\frac{3}{4}$ dozens of eggs, at $12\frac{1}{2}$ cts. a dozen?
3. What cost $13\frac{4}{5}$ bushels of turnips, at $37\frac{1}{2}$ cents a bushel?
4. What cost $10\frac{3}{5}$ yards of calico, at $15\frac{1}{2}$ cents a yard?
5. What cost $75\frac{1}{2}$ pounds of sugar, at $7\frac{3}{4}$ cts. a pound?
6. What cost $3\frac{4}{5}$ tons of hay, at $\$12\frac{3}{5}$ a ton?
7. What cost $6\frac{3}{4}$ bushels of apples, at $37\frac{1}{2}$ cents a bushel?
8. What cost $13\frac{1}{2}$ pounds of fish, at $9\frac{3}{4}$ cts. a pound?
9. What cost 175 pounds wool, at $39\frac{4}{5}$ cts. a pound?
10. What cost $18\frac{4}{5}$ yards of ribbon, at $23\frac{1}{2}$ cents a yard?
11. What cost 18 pocket handkerchiefs, at $\frac{3}{4}$ of a dollar each?
12. What cost $22\frac{3}{5}$ yards of selicia, at $87\frac{3}{4}$ cents a yard?
13. What cost $35\frac{4}{5}$ pounds of raisins, at $18\frac{3}{4}$ cents a pound?
14. What cost $75\frac{3}{4}$ bushels of wheat, at $\$1\frac{3}{5}$ a bushel?
15. What cost $23\frac{4}{5}$ cords of wood, at $\$3\frac{3}{8}$ a cord?
16. What cost $212\frac{2}{3}$ pounds of beef, at $7\frac{1}{2}$ cents a pound?
17. What cost $14\frac{4}{5}$ barrels of vinegar, at $\$10\frac{2}{5}$ a barrel?
18. What cost $22\frac{3}{4}$ barrels of sugar, at $\$15\frac{4}{5}$ a barrel?
19. What cost $35\frac{1}{2}$ tons of coal, at $\$9\frac{3}{4}$ a ton?

Division of Common Fractions.

ART. **129.** *Division of common fractions* is the method of dividing one fraction by another, or whole numbers and fractions by each other.

REMARK.—When the fractions have a common denominator, division can be performed by dividing the numerator of the one by the numerator of the other.

1. Divide $\frac{6}{9}$ by $\frac{2}{9}$.

OPERATION.

$$\tfrac{6}{9} \div \tfrac{2}{9} = 3 \text{ Ans.}$$

2. Divide $\frac{12}{17}$ by $\frac{6}{17}$.
3. Divide $\frac{9}{11}$ by $\frac{3}{11}$.

4. Divide $\frac{14}{17}$ by $\frac{7}{17}$.
5. Divide $\frac{9}{27}$ by $\frac{3}{27}$.
6. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

SOLUTION.—1 is contained in $\frac{2}{3}$, $\frac{2}{3}$ times; and if 1 is contained in $\frac{2}{3}$, $\frac{2}{3}$ times, $\frac{1}{4}$ is contained in $\frac{2}{3}$, $4 \times \frac{2}{3}$ times; and $\frac{3}{4}$ is contained in it $\frac{1}{3}$ of $4 \times \frac{2}{3}$ times $= \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$ times.

ART. **130.** Hence, to divide a fraction by a fraction, or fractions and whole numbers, by each other, we merely, *Invert the divisor and proceed as in multiplication, after having reduced compound fractions to simple ones, and whole and mixed numbers to improper fractions.*

7. Divide $\frac{4}{7}$ by $\frac{5}{6}$.
8. Divide $\frac{6}{8}$ by $\frac{4}{3}$.
9. Divide $2\frac{1}{3}$ by $\frac{4}{7}$.
10. Divide $3\frac{1}{4}$ by $1\frac{1}{2}$.
11. Divide $7\frac{1}{2}$ by $6\frac{1}{3}$.
12. Divide $8\frac{1}{4}$ by $6\frac{3}{5}$.
13. Divide $\frac{1}{3}$ of $\frac{3}{4}$ by $2\frac{3}{5}$.
14. Divide $\frac{2}{3}$ of $\frac{3}{5}$ by $2\frac{1}{2}$.
15. Divide $3\frac{1}{3}$ of $\frac{3}{5}$ by $\frac{3}{5}$ of $7\frac{1}{3}$.
16. Divide $4\frac{1}{5}$ times $3\frac{1}{7}$ by $8\frac{2}{3}$.
17. Divide $\frac{2}{5}$ of $\frac{10}{12}$ by $\frac{6}{7}$ of $\frac{14}{18}$.
18. Divide $\frac{9}{28}$ of $\frac{11}{12}$ of $8\frac{2}{7}$ by $4\frac{1}{5}$ times $6\frac{2}{7}$.
19. Divide $3\frac{1}{7}$ of $8\frac{1}{6}$ by $6\frac{1}{2}$ of $3\frac{1}{4}$:
20. Divide $\frac{7}{12}$ of $\frac{21}{25}$ of $8\frac{1}{3}$ times 4 by $\frac{5}{7}$ of $4\frac{1}{12}$.

PRACTICAL QUESTIONS IN DIVISION OF FRACTIONS.

1. At $\$\frac{2}{5}$ a bushel, how many bushels of apples can be bought for \$20?

2. At $\frac{7}{8}$ of a cent a piece, how many oranges can be bought for $14\frac{7}{8}$ cents?

3. If I pay $4\frac{2}{3}$ cents for riding 1 mile, how many miles can I ride for 280 cents?

4. A butcher expended $\$257\frac{1}{2}$ for sheep, at $\$1\frac{3}{4}$ a head; how many sheep did he buy?

5. How many pounds of tea, at $\$1\frac{3}{8}$ a pound, can be obtained for $\$19\frac{3}{8}$?

6. A lady bought $37\frac{2}{5}$ yards of calico for 561 cents; how much did it cost a yard?

7. A merchant bought 96 sheep for $\$99\frac{21}{25}$; how much did he give a head?

8. How many tons of coal, at $\$8\frac{3}{4}$ a ton, can be bought for $97?

9. A man paid $\$565\frac{1}{2}$ for a farm, giving $\$21\frac{3}{4}$ an acre; of how many acres did the farm consist?

10. At $\$1\frac{1}{5}$ a day, how many days must a man work for $\$31\frac{7}{10}$?

Complex Fractions.

Art. **131.** To reduce complex fractions to simple ones; we first, *Reduce compound fractions to simple ones, and whole and mixed numbers to improper fractions.* Then consider the denominator of the complex fraction a divisor and proceed as in division of fractions.

1. Reduce $\frac{\frac{3}{4}}{\frac{3}{5}}$ to a simple fraction.

OPERATION.

$$\frac{\frac{3}{4}}{\frac{3}{5}} = \frac{\cancel{3}}{4} \times \frac{5}{\cancel{3}} = \frac{5}{4} = 1\tfrac{1}{4}. \text{ Ans.}$$

2 Reduce $\frac{\frac{1}{2}}{\frac{1}{3}}$ to a simple fraction.

3. Reduce $\frac{\frac{2}{3}}{\frac{3}{4}}$ to a simple fraction.

4. Reduce $\frac{\frac{1}{2} \text{ of } \frac{2}{3}}{\frac{6}{8}}$ to a simple fraction.

5. Reduce $\frac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{3} \text{ of } \frac{3}{4}}$ to a simple fraction.

6. Reduce $\frac{5\frac{1}{2}}{\frac{1}{3} \text{ of } \frac{3}{4}}$ to a simple fraction.

7 Reduce $\frac{5\frac{2}{3}}{\frac{7}{8} \text{ of } 5\frac{1}{2}}$ to a simple fraction.

8 Reduce $\dfrac{2\frac{1}{2} \text{ of } 3\frac{1}{3}}{7\frac{2}{3} \text{ of } 3\frac{3}{4}}$ to a simple fraction.

9. Reduce $\dfrac{\frac{1}{2} + \frac{3}{4}}{\frac{2}{3} - \frac{\ }{4}}$ to a simple fraction.

10. Reduce $\dfrac{\frac{2}{3} + \frac{4}{5} \text{ of } \frac{5}{6}}{\frac{3}{4} \text{ of } \frac{4}{7} + \frac{2}{5}}$ to a simple fraction.

LEAST COMMON MULTIPLE OF FRACTIONS.

ART. **132.** The *Least Common Multiple* of any two or more fractions, is the smallest number that will, when divided by each of them, give an *integer* for a quotient.

Since, in dividing any number by a fraction, the denominator of that fraction becomes a *multiplier*, and the numerator a *divisor* of that number; it is evident,—*That the least common multiple of any two or more fractions*, after reducing *mixed numbers to improper fractions, compound fractions to simple ones*, and all to their *lowest terms*, will be the *quotient arising from dividing the least common multiple of their numerators by the greatest common measure of their denominators.*

1. What is the least common multiple of $4\frac{54}{84}$, $5\frac{12}{112}$, and $2\frac{66}{84}$?

SOLUTION.—The above mixed numbers, changed to improper fractions become $\frac{390}{84}$, $\frac{572}{112}$, and $\frac{234}{84}$. These fractions when reduced to their lowest terms become, $\frac{65}{14}$, $\frac{143}{28}$, and $\frac{117}{42}$. The least common multiple of the numerators, (65, 143, and 117) of these fractions is 6435. See Art. 94.

The greatest common measure of the denominators, (14, 28, and 42,) of these fractions is 14. See Art 89. Hence, 6435, the least common multiple of the numerators of these fractions, is 14 times larger than the least common multiple of the fractions. Consequently $\frac{6435}{14}$ or $459\frac{9}{14}$ is the least common multiple required.

2. What is the least common multiple of $4\frac{4}{15}$, $7\frac{1}{5}$ and $2\frac{2}{35}$?

3. What is the least common multiple of $4\frac{2}{7}$, $9\frac{5}{14}$, and $12\frac{1}{7}$?

* NOTE.—The student will readily observe that the GREATEST COMMOM MEASURE of any two or more fractions, after being reduced to their simplest form, will be the QUOTIENT arising from dividing the *greatest common measure* of their numerators by the *least common multiple* of their denominators.

What is the greatest common measure of $\frac{7}{9}$, $\frac{14}{15}$, and $1\frac{13}{15}$?

Ans. $\frac{7}{45}$.

4. What is the least common multiple of $\frac{10}{12}$, $\frac{24}{48}$, $\frac{18}{27}$, and $\frac{25}{75}$?

5. What is the least common multiple of $2\frac{4}{8}$, $16\frac{4}{6}$, and $10\frac{6}{12}$?

PRACTICAL QUESTIONS IN MULTIPLES.

1. What is the smallest sum of money for which a person could purchase, either a number of geese, at $\$1\frac{1}{2}$ a piece; or a number of turkeys, at $\$2\frac{1}{4}$ a piece, and how many of each could be bought,—the entire sum to be employed in either purchase?

2. A can travel $6\frac{2}{5}$ miles in a day; B $11\frac{3}{15}$ miles; C $20\frac{5}{20}$ miles; and D $30\frac{4}{25}$ miles in a day. What is the least number of miles that will afford a number of whole days' travel for any of the four, and how many days would it take each to accomplish the journey?

3. What is the least number of bushels of grain that will fill a number of hogsheads, each containing $10\frac{5}{45}$ bushels; a number of boxes, each containing $23\frac{25}{30}$ bushels; or a number of bins, each containing $25\frac{10}{15}$ bushels; and how many times would it fill each of them?

4. What is the smallest sum of money for which I could purchase a number of cows, at $\$20\frac{10}{45}$; a number of oxen, at $\$47\frac{20}{30}$; or a number of horses, at $\$51\frac{5}{15}$; and what number of each could I purchase for that sum?

5. Three vessels A, B, and C start from the same place, at the same time, and sail in the same direction around an island 30 miles in circumference; A at the rate of 3, B 11, and C 23 miles an hour. How many hours before they will all meet at the place from which they started? How many hours before they will first meet, and at what point? Suppose they continue sailing, how often will they all be together?

Solution.—A moves at the rate of 3 miles an hour; consequently, to move 1 mile it will take $\frac{1}{3}$ of an hour, and to move 30 miles, (once around the island,) it will take $\frac{30}{3}$ or 10 hours. In a similar way, we find B will move

once around the island in $\frac{30}{11}$ of an hour; and C, in $\frac{30}{23}$ of an hour. Now it is evident that the least common multiple of 10, $\frac{30}{11}$, and $\frac{30}{23}$ will express the number of hours that must elapse before they will all meet at the place from which they started; which is 30 hours.

How long before they will first meet? C gains on B, $23 - 11 = 12$ miles in 1 hour; consequently, to gain 1 mile it will take $\frac{1}{12}$ of an hour, and to gain 30 miles, (the distance he must gain before he overtakes B,) 30 times $\frac{1}{12} = \frac{30}{12}$, or $\frac{5}{2}$ *of an hour.*—B gains on A $11 - 3 = 8$ miles in 1 hour; hence to gain 1 mile it will take $\frac{1}{8}$ of an hour, and to gain 30 miles, (the distance he must gain before he overtakes A,) 30 times $\frac{1}{8} = \frac{30}{8}$, or $\frac{15}{4}$ *of an hour.* Since C will overtake B in $\frac{5}{2}$ of an hour; and B will overtake A in $\frac{15}{4}$ of an hour; it is evident that the number of hours that must elapse before C will overtake B, at the same time that B overtakes A will be the least common multiple of $\frac{5}{2}$ and $\frac{15}{4}$, which is $7\frac{1}{2}$ hours.

If they are together in $7\frac{1}{2}$ hours,

A must sail	$22\frac{1}{2}$	miles,	or 0	times around the island	+ $22\frac{1}{2}$	miles.
B " "	$82\frac{1}{2}$	"	or 2	" " " "	+ $22\frac{1}{2}$	"
C " "	$172\frac{1}{2}$	".	or 5	" " " "	+ $22\frac{1}{2}$	"

Consequently $22\frac{1}{2}$ miles from the place from which they started is the place where they first will be together.

If they contiuue traveling they will be together every $7\frac{1}{2}$ hours.

6. If four men A, B, C, and D, start from the same place at the same time, and walk around an island 27 miles in circumference; A at the rate of 4, B 12, C 20, and D 28 miles a day; how many miles will each have to travel before they meet, and how many days before they will all meet at the place from which they started?

7. There are three wheels, A, B, and C, each $10\frac{2}{7}$ feet in circumference, standing with their axes in a right line, with a letter M on the circumference of each, which are also in a right line. If these wheels be set in motion; A at the rate of $5\frac{1}{7}$, B $7\frac{1}{5}$, and C $13\frac{1}{3}$ feet in 1 second, how long before the M's. on the circumference of the wheels

will all be in a right line again, and how many revolutions will each have made ?

PRACTICAL QUESTIONS IN FRACTIONS.

1. Reduce $231\frac{8}{17}$ to an improper fraction.

2. Reduce $478\frac{23}{141}$ to an improper fraction.

3. Reduce $\frac{1864}{12}$ to a mixed number.

4. Reduce $\frac{2768}{17}$ to a mixed number.

5. Reduce $\frac{750}{970}$ to its lowest terms.

6. Reduce $\frac{22736}{25984}$ to its lowest terms.

7. Reduce $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{9}{18}$ of $\frac{16}{24}$ of $\frac{16}{21}$ to its simplest form.

8. Reduce $\frac{36}{28}$ of $\frac{15}{45}$ of $\frac{56}{96}$ of $\frac{24}{48}$ to its simplest form.

9. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{4}{7}$ to equivalent fractions, having a common denomination.

10. Reduce $\frac{8}{12}$, $\frac{15}{16}$ and $\frac{17}{48}$ to equivalent fractions, having the least common denomination.

11. What is the sum of $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{11}{12}$?

12. What is the sum of $\frac{8}{10}$ of $4\frac{1}{2}$, $\frac{5}{6}$ of $3\frac{2}{3}$, and $\frac{3}{7}$ of $6\frac{1}{6}$?

13. From $8\frac{3}{4}$ subtract $6\frac{2}{7}$.

14. From $\frac{2}{3}$ of $8\frac{1}{2}$ subtract $\frac{3}{4}$ of $2\frac{3}{4}$.

15. Divide $3\frac{1}{2}$ by $4\frac{1}{3}$.

16. Divide $\frac{1}{2}$ of $2\frac{2}{3}$ by $\frac{2}{3}$ of $4\frac{4}{5}$.

17. A has $4\frac{3}{5}$ times \$25, and B has $2\frac{3}{7}$ times \$$8\frac{2}{3}$; how much more has A than B.

18. A has $\frac{3}{8}$ of \$16; B $\frac{8}{9}$ of \$473; C $\frac{4}{5}$ of \$$862\frac{1}{2}$; and D $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of \$$168\frac{1}{2}$. How many dollars have they together.

19. A had $\frac{2}{3}$ of $\frac{9}{8}$ of $12\frac{1}{2}$ times \$$8643\frac{1}{2}$, and paid $\frac{4}{5}$ of $\frac{1}{8}$ of it for a farm; how much had he remaining ?

20. A had \$86472, which was $6\frac{3}{5}$ times as much as B had; how much had B ?

21. A has 1278 sheep, which is 188 more than $\frac{2}{3}$ of $3\frac{1}{3}$ times B's number; how many sheep has B ?

22. A and B own 680 acres of land; $\frac{2}{3}$ of A's number of acres equals $\frac{3}{4}$ of B's;—how many acres have each ?

23. A farmer has 495 bushels of wheat and rye together,

and $\frac{3}{4}$ of the number of bush. of wheat equals $\frac{6}{7}$ of the number of bush. of rye. How many bushels of each has he?

24. A speculator bought 688 geese and turkeys; how many of each did he buy, providing there were only $\frac{3}{5}$ as many geese as turkeys?

25. A and B together own 824 sheep; how many has each, providing A has $1\frac{2}{3}$ times as many as B?

26. A owns $\frac{3}{17}$ of a certain tract of land, containing 98600 acres; B owns $\frac{15}{29}$ of the remainder; C owns $\frac{3}{10}$ as much as A and B together; and D owns the remainder. How much does each own?

27. A merchant expended \$463 for dry goods; $\frac{2}{3}$ of $\frac{9}{14}$ of the remainder for groceries; and what then remained, which was \$4680, he expended for a store and lot. How much did the groceries cost?

28. A gentleman invested $\frac{2}{5}$ of his fortune in speculation, and the remainder, which was \$1630 more than the half of his fortune, he put out on interest. At the end of the year he gained by speculation $\frac{5}{16}$ as much as he laid out, and his interest was $\frac{2}{10}$ of the principal; how much was his fortune, and how much did he gain during the year?

29. A man being asked the value of his horse, replied, that its value increased by its $\frac{3}{7}$ and \$$268\frac{2}{3}$ more, equaled \$864. What was the value of the horse?

30. A farmer has $\frac{3}{7}$ of the number of his sheep in one field; and the remainder, which is 46 more than the half of his flock in a second field. How many sheep has he in each field, and how many in both?

31. A certain sum of money was divided between two brothers, James and Jackson; James took $\frac{3}{5}$ of it, lacking \$145; and Jackson the remainder. It now appears that each has the same sum. How much did each receive?

32. From a certain flock of sheep A purchased $\frac{2}{7}$ of them; B $\frac{2}{9}$ of them; C $\frac{2}{5}$ of them; and D the remainder, which was 116. How many sheep were then in the field, and how many did A, B, and C, buy respectively?

33. A has $\frac{2}{3}$ of $1\frac{5}{7}$ times \$2660 which is $3\frac{5}{7}$ times as many again as B has. How many dollars has B?

34. A and B together own a farm? A owns $\frac{7}{15}$ of it; and B $\frac{8}{15}$ of it. If B should sell to A $12\frac{1}{2}$ acres, they would then each have the same number of acres. How many acres has each?

35. An estate was divided among A, B, and C. A had $\frac{3}{7}$ of it; B $\frac{1}{3}$ of it; and C the remainder. A, by this division, received $180 more than B. How much was the estate, and how much did each receive?

36. Divide $9926 among A, B, C, and D, so that A shall have $\frac{3}{7}$ of it, lacking $1812; B $\frac{1}{2}$ of the remainder, lacking $858; C $\frac{4}{5}$ of what now remains, lacking $1380; and D what is left?

37. A gentleman's house cost $4800, and $1\frac{3}{4}$ times its cost, is $3\frac{1}{3}$ times $\frac{6}{7}$ of the cost of the furniture contained in it; what was the cost of the furniture?

38. Henry had $\frac{2}{7}$ of a certain fortune; Perry $\frac{9}{13}$ of it; and Elisha the remainder, which was $1600. How much was the fortune, and how much did Henry and Perry receive respectively?

39. Bought at one time 460 acres of land at $$25\frac{1}{2}$ an acre; at another time 345 acres, at $$43\frac{1}{3}$ an acre. If $\frac{3}{5}$ of the whole quantity were sold, at $21 an acre, and the remainder, at $34 an acre, what would be the gain or loss?

40. A merchant purchased 120 yards of cloth for $780, and sold $\frac{5}{6}$ of it at a profit of $$1\frac{3}{8}$ a yard; and the remainder, at a loss of $$\frac{5}{8}$ a yard. How much did he gain by the operation?

41. A person bought 38 barrels of flour at $$4\frac{1}{2}$ a barrel. Having sold $21\frac{1}{2}$ barrels of them, at $$4\frac{3}{4}$ a barrel, at what price a barrel must the remainder be sold to gain $$25\frac{1}{2}$ on the whole.

42. James, Henry, and Joseph were employed to hoe a field of corn for $32.10. James could hoe a row in $20\frac{4}{5}$ minutes; Henry in $25\frac{3}{5}$ minutes; and Joseph in $32\frac{8}{10}$ minutes. It so happened that when they all first came to the end of a row at the same instant, that the work was completed. How long were they engaged in the field; how many rows did the field contain; and how much in equity ought each to receive?

CHAPTER VI.

DECIMAL FRACTIONS.

ART. **133.** A DECIMAL FRACTION is one in which the denominator is not expressed, but is understood to be a unit followed by one or more ciphers; or such a fraction the successive orders of which increase from right to left in a *tenfold* ratio, consequently decrease from left to right in the same ratio.

Decimal Fractions originate from dividing 1 into 10 equal parts; each of these parts into 10 other equal parts; and each of the parts thus obtained into 10 other equal parts, and so on, indefinitely. Thus, $1 \div 10 = \frac{1}{10}$; $\frac{1}{10} \div 10 = \frac{1}{100}$; $\frac{1}{100} \div 10 = \frac{1}{1000}$, &c., which are expressed in Decimals as follows:—

$$\tfrac{1}{10} = \cdot 1;\ \tfrac{1}{100} = \cdot 01;\ \tfrac{1}{1000} = \cdot 001, \text{ \&c.}$$

ART. **134.** In expressing DECIMAL FRACTIONS, the numerator only is written with a point before it, called a *Decimal point* or *Separatrix*, to distinguish it from whole numbers; the denominator being understood. Thus,

$\frac{7}{10} = \cdot 7$ tenths.
$\frac{7}{100} = \cdot 07$ hundredths.
$\frac{7}{1000} = \cdot 007$ thousandths.
$\frac{7}{10000} = \cdot 0007$ ten-thousandths.
$\frac{223}{10000} = \cdot 0223$ ten-thousandths.

By inspecting the above fractions, it is observed that *tenths* occupy the *first* place at the right of the *decimal point*; that *hundredths* occupy the *second* place; that *thousandths* occupy the *third* place, &c.

We also observe that each removal of a figure one place towards the right, decreases its value in a *tenfold* ratio. Hence,

Every cipher placed on the left of a decimal figure diminishes its value in a tenfold ratio. Thus, $\cdot 9 = \frac{9}{10}$; $\cdot 09 = \frac{9}{100}$; and $\cdot 009 = \frac{9}{1000}$, &c.

If a cipher be placed on the right of a decimal figure, it does not change its value, as the figure still occupies the same place. Thus, $\cdot 9 = \cdot 90 = \cdot 900$, $\frac{9}{10} = \frac{90}{100} = \frac{900}{1000}$, &c.

NUMERATION OF DECIMAL FRACTIONS.

ART. **135.** A whole number and a decimal fraction, when considered together, is called a *mixed number;* the relation and names of which can be learned from the following

TABLE.

&c., &c.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point* or *Separatrix.*	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.	Billionths.	Ten Billionths.	Hundred Billionths.	&c., &c.
	9	3	1	4	.	3	3	3	3	3	4	7	8	6	2	7	
	Whole Numbers.					Decimals.											

ART. **136.** To read a Decimal number expressed in figures.

Read the figures as in whole numbers, and add the name of the decimal place. Thus, ·9 is read *nine-tenths;* ·09 is read *nine hundredths;* ·100024 is read *one hundred thousand and* 24 *millionths*, &c

REMARK.—To ascertain the name of the right hand figure, begin at the left and name each figure till you come to the last one, which will be the name required.

Read the following numbers.

1.	·13	6.	46·824	11.	102·460203
2.	·06	7.	36·4231	12.	14683·04000602
3.	·0102	8.	67·46212	13.	18602·84683002
4.	·02202	9.	147·8630202	14.	10001·861320401
5.	·060708	10.	1468·70002	15.	1010101·10101010101

UNITED STATES CURRENCY,* OR FEDERAL MONEY.

ART. **137.** United States Currency is the legal *Money* of the United States, and is expressed according to the *Decimal Scale* of notation.

Money may be considered the measure of the value of things.

† The denominations of the United States Currency are *Eagles*, *Dollars*, *Dimes*, *Cents*, and *Mills*.

The coins of the United States are the

Double-Eagle,
Eagle,
Half-Eagle,
Quarter-Eagle,
and Dollar, } made of gold,

The Dollar,
Half-Dollar,
Quarter-Dollar,
Dime,
Half-Dime,
and Three-cent piece, } made of silver;

The cent is made of copper.
The mill is not coined.

NOTE.—By an act of Congress, January 18th, 1837, the gold and silver coin must consist of $\frac{9}{10}$ pure metal, and $\frac{1}{10}$ alloy.

REMARK.—* This currency is usually called *Federal Money*, because it was made the currency of the United States at the time of the adoption of the *Federal Constitution*, August 8th, 1786.

† The word *Dollar* is derived from a Danish word, which was derived from *Dale*, the name of the town in which this coin was first made. The word *Dime* is derived from a French word signifying *ten*;—the word *Cent*, from a Latin word signifying *one hundred*;—the word *Mill*, from a Latin word signifying *one thousand*. The terms, *Dime*, *Cent*, and *Mill*, are applied to coins of our currency, in consequence of the relation they respectively bear to the DOLLAR.

The alloy for gold must consist of an equal quantity of silver and copper, and the alloy for silver of pure copper.

The three-cent piece is $\frac{3}{4}$ silver and $\frac{1}{4}$ copper.

Before 1837, the gold for coinage consisted of $\frac{22}{24}$ pure gold, $\frac{1}{24}$ silver, and $\frac{1}{24}$ copper; or, as sometimes expressed, 22 *carats* gold, 1 of silver, and 1 of copper; the word *carat* meaning one twenty-fourth.

Silver for coinage consisted of 1489 parts of pure silver, and 179 parts of pure copper; or, expressed in carats, $21\frac{59}{139}$ of silver, and $2\frac{80}{139}$ of copper.

TABLE OF UNITED STATES CURRENCY.

10 Mills	make	1 Cent,	marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$.*
10 Dollars	"	1 Eagle,	"	E.

ART. **138.** The accounts of the United States are kept in *Dollars Cents*, and *Mills*. In business transactions, the eagle is expressed in dollars,—the dime in cents. Thus, 4 eagles, according to the preceding table, is = 40 dollars; and, instead of saying 4 eagles and 5 dollars, we say 45 *dollars*. Also, 7 dimes are = 70 cents; therefore, instead of saying 7 dimes and 9 cents, we say 79 cents, &c.

ART. **139.** In the United States Currency, *dollars* are *Integers*, and therefore, occupy the place of units. Cents express hundredths of a dollar, consequently they occupy the first and second place on the right of the decimal point; the third place is mills; &c. Thus, $47·356, is read forty-seven dollars thirty-five cents and six mills.

ART. **140.** To express any number of cents less than 10, there should be a cipher placed between them and the decimal point, as cents are hundredths of a dollar, and therefore occupy two places; if mills only are expressed,

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* This symbol is probably a contraction of the letter U, placed upon an S, to denote U. S. (United States.)

two ciphers should be placed between them and the decimal point. Thus:

4 cents is written \$·04, and is read 4 *hundredths* of a dollar. $12\frac{1}{2}$ cents is written \$·$12\frac{1}{2}$, and is read $12\frac{1}{2}$ *hundredths* of a dollar. $\frac{3}{4}$ of a cent is written \$·$00\frac{3}{4}$, and is read $\frac{3}{4}$ of 1 *hundredths* of a dollar. 5 mills is written \$·005, and is read 5 *thousandths* of a dollar. 4 cents and 6 mills is written \$·046, and is read 46 *thousandths* of a dollar.

ART. **141.** It might be well to observe, that the first figure after the decimal point, expresses *tenths* of a dollar, or *tens of cents;* the second, cents; the first two places taken together express *hundredths* of a dollar, or *cents;* the third *mills;* the fourth *tenths* of mills, &c.

REDUCTION OF DECIMALS TO COMMON FRACTIONS.

ART. **142.** From what has been said of Decimal Fractions, we infer that a decimal can be reduced to a common fraction by *erasing the decimal point, and underneath writing the denominator*, which is a unit followed by as many ciphers as there are places in the decimal; *then reduce the fraction to its lowest terms.*

1. Reduce ·025 to a common fraction.

OPERATION.

$$\cdot 025 = \frac{025}{1000} = \frac{1}{40}. \text{ Ans.}$$

Reduce each of the following decimals of a dollar to equivalent common fractions.

2. \$·0625.	5. \$·25.	8. \$·625.
3. \$·125.	6. \$·5.	9. \$·875.
4. \$·375.	7. \$·75.	10. \$·5625.

11. Express 6·0625 by an integer and a common fraction.

12. Express 12·25 by an integer and a common fraction

13. Express 14·5 by an integer and a common fraction

14. Express 25·625 by an integer and a common fraction.

15. Express 45·875 by an integer and a common fraction.

16. Express 37·75 by an integer and a common fraction.

17. Express 16·375 by an integer and a common fraction.

18. Express 34·9375 by an integer and a common fraction.

19. Express 62·5625 by an integer and a common fraction.

20. Express 141·1875 by an integer and a common fraction.

Reduction of Common Fractions to Decimals.

1. Reduce $\frac{4}{7}$ to a decimal.

OPERATION.

	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hund. Thousandths.	&c. &c.
7)	4·	0	0	0	0	0	
	0·	5	7	1	4	2+*	

EXPLANATION.—7 is not contained into 4 units a whole number of times, therefore, we reduce 4 to *tenths;* 4 = 40 *tenths.* 7 is contained in 40 tenths, 5 *tenths* times and 5 tenths remaining,—which equals 50 *hundredths.* 7 is contained in 50 hundredth, 7 *hundredths* times, and 1 *hundredth* remaining, which equals 10 *thousanths*, &c.

REMARK.—If the decimal is carried out six places, the result will be sufficiently exact for all practical purposes.

What decimal of a dollar is equal to the following fractions of a dollar?

1. $\$\frac{1}{4}$.
2. $\$\frac{1}{3}$.
3. $\$\frac{1}{2}$.
4. $\$\frac{3}{4}$.
5. $\$\frac{3}{8}$.
6. $\$\frac{5}{8}$.
7. $\$\frac{7}{8}$.

8. Reduce $\$12\frac{1}{2}$ to an equivalent decimal expression

9. Reduce $\$16\frac{1}{3}$ to an equivalent decimal expression.

* The symbol, +, when placed after a decimal indicates more.

10. Reduce $\$42\frac{1}{8}$ to an equivalent decimal expression.
11. Reduce $\$25\frac{5}{16}$ to an equivalent decimal expression
12. Reduce $\$18\frac{7}{16}$ to an equivalent decimal expression.
13. Reduce $\$37\frac{13}{16}$ to an equivalent decimal expression.

Reduction of Mixed Decimals to Simple Decimals.

ART. **143.** If, in the place of the Common Fraction, we place its equivalent decimal without the point prefixed, the value remains the same. Thus, $.4\frac{3}{4}$.

$$\text{The } \tfrac{3}{4} = {\cdot}75, \text{ hence } {\cdot}4\tfrac{3}{4} = {\cdot}475$$

14. Reduce $\cdot25\frac{1}{8}$ to an equivalent simple decimal.
15. Reduce $\cdot27\frac{1}{4}$ to an equivalent simple decimal.
16. Reduce $\cdot35\frac{1}{2}$ to an equivalent simple decimal.
17. Reduce $\cdot47\frac{3}{4}$ to an equivalent simple decimal.
18. Reduce $\cdot64\frac{4}{5}$ to an equivalent simple decimal.
19. Reduce $\cdot84\frac{3}{5}$ to an equivalent simple decimal.
20. Reduce $\cdot146\frac{2}{5}$ to an equivalent simple decimal.

Repetends.

ART. **144.** If a common fraction cannot be accurately expressed in decimals, it is evident that the decimal figures will recur in periods; and that the number of figures in the period cannot exceed the number of units in the denominator, less one—for every remainder must be less than the denominator; and whenever a remainder occurs like one previously obtained, the decimal figures will begin to repeat.

Decimals that recur in this way are called *repeating decimals;* the figures repeated are called a *repetend*, and are distinguished by a ($\dot{}$) placed over the first and last, as, $\frac{1}{3} = {\cdot}333$, &c. $= {\cdot}\dot{3}$; $\frac{1}{7} = {\cdot}142857142$, &c. $= {\cdot}\dot{1}4285\dot{7}$.

If decimal figures precede the repetend, they are called the *finite part* of the decimal; as, $\frac{1}{12} = 0{\cdot}0833$, &c. The *finite part* is $\cdot08$.

When the repeating period begins with the first decimal figure, it is called a simple repetend.

A simple repetend that contains as many figures in the repeating part as there are units in the denominator, less one, is called a *Perfect Repetend.* Thus,

$$\tfrac{1}{7} = 0{\cdot}\dot{1}4285\dot{7}.$$
$$\tfrac{1}{17} = 0{\cdot}\dot{0}58823529411764\dot{7}.$$
$$\tfrac{1}{19} = 0{\cdot}\dot{0}5263157894736842\dot{1}.$$

&c. &c.

Compound Repetends.

The following fractions are called *Compound Repetends,* because they consist of a *finite* and a repeating part.

$$\tfrac{1}{6} = 0{\cdot}1\dot{6}.$$
$$\tfrac{1}{12} = 0{\cdot}08\dot{3}.$$
$$\tfrac{1}{14} = 0{\cdot}0\dot{7}1428\dot{5}.$$

&c. &c.

Art. 145. Repetends Reduced to Common Frations.

$${\cdot}\dot{1} = \tfrac{1}{9};\ {\cdot}\dot{0}\dot{1} = \tfrac{1}{99};\ {\cdot}\dot{0}0\dot{1} = \tfrac{1}{999};\ {\cdot}\dot{0}00\dot{1} = \tfrac{1}{9999}.$$
$${\cdot}\dot{2} = \tfrac{2}{9};\ {\cdot}\dot{0}\dot{2} = \tfrac{2}{99};\ {\cdot}\dot{0}0\dot{2} = \tfrac{2}{999};\ {\cdot}\dot{0}00\dot{2} = \tfrac{2}{9999}.$$
$${\cdot}\dot{3} = \tfrac{3}{9};\ {\cdot}\dot{0}\dot{3} = \tfrac{3}{99};\ {\cdot}\dot{0}0\dot{3} = \tfrac{3}{999};\ {\cdot}\dot{0}00\dot{3} = \tfrac{3}{9999}.$$

&c., &c., &c., &c.

From which we observe that the repetend, with the decimal point and useless ciphers on the left, erased, is the numerator, and the denominator is as many 9's as there are places in the repetend.

Thus, ·004004, &c. $= \tfrac{4}{999}$.

1. Redcce $\cdot\dot{1}4285\dot{7}$ to a common fraction.

2. Reduce $\cdot\dot{0}2\dot{7}$ to a common fraction.

3. Reduce $\cdot\dot{0}1\dot{4}$ to a common fraction.

4. Reduce $\cdot\dot{7}\dot{2}$ to a common fraction.

5. Reduce $\cdot\dot{1}2\dot{3}$ to a common fraction.

6. Reduce $\cdot\dot{2}3809\dot{5}$ to a common fraction.

ART. **146.** REDUCTION OF COMPOUND REPETENDS

7. Reduce ·083333, &c., to a common fraction.

OPERATION.

$$\cdot083333, \text{\&c.} = \cdot08\dot{3} = \cdot08\tfrac{3}{9} = \frac{8\frac{1}{3}}{100} = \frac{25}{300} = \frac{1}{12}. \text{ Ans.}$$

8. Reduce $\cdot1\dot{6}$ to a common fraction.

9. Reduce $\cdot0\dot{6}$ to a common fraction.

10. Reduce $\cdot0\dot{4}\dot{5}$ to a common fraction.

11. Reduce $\cdot041\dot{6}$ to a common fraction.

12. Reduce $\cdot0\dot{7}1428\dot{5}$ to a common fraction.

ART. **147.** We have seen that the value of some common fractions can be accurately expressed in decimals, while that of others can *only* be approximately expressed, as the process of division will never terminate.

As we annex ciphers to the numerator and divide by the denominator to change a *common* to a *decimal* fraction; and, as annexing a cipher to any number is the same as multiplying it by 10, it follows that whenever the prime factors of the denominator of a common fraction (after reducing it to its lowest terms) do not differ from 2 and 5 (the prime factors of 10), the division will terminate.

Hence, to determine whether a common fraction can be accurately expressed in decimals,

Reduce it to its lowest terms, then resolve the denominator into its prime factors; if these factors do not differ from 2 *and* 5 (*the factors of* 10), *the fraction can be accurately expressed in decimals.* It is also evident that *the highest exponent of the* 2 *or* 5 *will denote the number of decimal places required to express it.*

1. Can $\frac{3}{625}$ be accurately expressed in decimals? If so, how many places will be required to express it?

2. Can $\frac{9}{160}$ be accurately expressed in decimals? If so, how many places will be required to express it?

3. Can $\frac{7}{128}$ be accurately expressed in decimals? If so, how many places will be required to express it?

4. Can $\frac{7}{36}$ be accurately expressed in decimals? If so, how many places will be required to express it?

5. Can $\frac{7}{750}$ be accurately expressed in decimals?

Addition of Decimals and United States Currency.

Art. **148.** Since decimals increase from right to left, and decrease from left to right, in the same ratio as simple numbers, we can *add*, *subtract*, *multiply*, or *divide* them, in the same manner as though they were abstract numbers. In addition, observe to place units under units, tens under tens, &c. Hence the decimal points will always come one under another.

1. What is the sum of 14·623, 231·6231, 101·36, 8002·68023, and 7·462312?

OPERATION.

Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	
		1	4·	6	2	3				
	2	3	1·	6	2	3	1			
	1	0	1·	3	6					
8	0	0	2·	6	8	0	2	3		
			7·	4	6	2	3	1	2	
8	3	5	7·	7	4	8	6	4	2	Ans.

2. What is the sum of 12·6, 63·04, 342·021, 8462·7321 62·48132, and 472·16321?

3. What is the sum of \$27·046, \$14·4041, \$7·86321, \$324·8631, and \$4123·82631?

4. What is the sum of 8·321, 9·6231, 84·7163, 116·84324, and 18312.83201?

5. What is the sum of 82·631, 1·7632, 8413·0001, 732·4671, 842·71, and 9·802?

PRACTICAL QUESTIONS.

1. Sold a box of candles for \$12·25; a barrel of flour for \$7·75; a sack of coffee for \$12·12$\frac{1}{2}$; and a barrel of sugar for \$18·37$\frac{1}{2}$; required the amount I should receive.

2. What cost a horse, a yoke of oxen, a cow, and a sheep, if the horse cost \$147·62$\frac{1}{2}$; the oxen, \$184·06$\frac{1}{4}$; the cow, \$46·82; and the sheep, \$7·06$\frac{1}{4}$?

3. A merchant bought broadcloth to the amount of \$872·45; muslin and linen to the amount of \$184·75; sugar to the amount of \$296·85; and flour to the amount of \$387·80. What was the whole cost?

4. A gentleman has some young cattle worth \$6782·87; a horse worth \$285·60; a yoke of oxen worth \$196·87; four cows worth \$210·20; and a farm worth \$8642·84. What is the value of the whole?

5. A merchant bought a quantity of goods for \$89785·95; paid for duties \$897·40; and for transportation \$388·87. For how much must he sell the goods to gain \$346·82?

6. Bought a ton of hay, for \$14·87$\frac{1}{2}$; a cord of wood, for \$6·12$\frac{1}{2}$; a barrel of apples, for \$3·06$\frac{1}{4}$; a barrel of flour, for \$8·37$\frac{1}{2}$; and a quarter of beef, for \$9·87$\frac{1}{2}$. Required the sum to be paid?

7. A farmer's bill at the store was as follows: 4$\frac{1}{3}$ yards of cloth, \$24·12$\frac{1}{2}$; 3 pair of boots, \$16·87$\frac{1}{2}$; a dozen skeins of silk, \$·87$\frac{1}{2}$; and 15 yards of muslin, \$1·12$\frac{1}{2}$. Required the amount of the bill?

8. A farmer sold produce as follows: wheat, for \$325·87$\frac{1}{2}$; corn, for \$137·62$\frac{1}{2}$; rye, for \$237·85; oats, for \$96·06$\frac{1}{4}$; hay, for \$62·62$\frac{1}{2}$. Required the amount of the sale?

9. What should be paid for a barrel of sugar, worth \$18·47$\frac{1}{2}$; a quarter of beef, worth \$9·87$\frac{1}{2}$; a barrel of flour, worth \$6·37$\frac{1}{2}$; a box of raisins, worth \$8·20; a fir

kin of butter, worth $15·97\frac{1}{2}$; and a barrel of molasses, worth $12·25?

10. Bought a quantity of sugar, for $183·92; a quantity of flour, for $227·62\frac{1}{2}$; a quantity of hams, for $384·18\frac{3}{4}$; a quantity of corn, for $386·87\frac{1}{2}$. For how much must it all be sold so as to gain $465·85, after paying $120·37\frac{1}{2}$ for cartage and storage?

SUBTRACTION OF DECIMALS AND THE UNITED STATES CURRENCY.

1. From 64·5 subtract 37·8046.

OPERATION.

Min.	64·5000
Sub.	37·8046
Rem.	26·6954

REMARK.—In examples of this kind we annex ciphers to the minuend, which does not effect its value. (See last paragraph, Art. 134.)

Care must be taken to place the numbers so that the decimal points shall stand one under another, in order that units may be taken from units, &c.; tenths from tenths, &c.

2. From 204·614 subtract 9·131.
3. From 6 subtract 4·00006.
4. From 4·4 subtract 3·00004.
5. From 1 subtract ·000001.
6. From 16802·4682 subtract 981·8364.
7. Subtract 10014·40001 from 80084·600861.

PRACTICAL QUESTIONS.

1. A man bought a span of horses for $465·85, and a yoke of oxen for $195·38; how much more did he pay for the horses than for the oxen?

2. A gentleman having $18654·84, gave $2685·69 of it for a store; how much money has he remaining?

3 A man is owing $6785·95, and has due him $9986·125; how much more is due him than what he owes?

4. A quantity of lumber was bought for $5682·18$\frac{3}{4}$, and sold for $7631·56$\frac{1}{4}$; how much was the gain?

5. A quantity of flour was purchased for $\$3896{\cdot}12\frac{1}{2}$, and sold for $\$3509{\cdot}18\frac{3}{4}$; how much was the loss?

6. A grazier bought cattle for $\$384{\cdot}95$, and sheep for $\$135{\cdot}68$. He sold the cattle for $\$479{\cdot}12\frac{1}{2}$, and the sheep for $\$109{\cdot}72\frac{1}{2}$; how much did he gain by these transactions?

7. A manufacturer purchased a quantity of cotton for $\$387{\cdot}95$, which he made into cloth, at an expense of $\$184{\cdot}06\frac{1}{4}$; how much will he make by selling the cloth for $600?

8. A speculator purchased wheat for $\$587{\cdot}87\frac{1}{2}$, and pork for $\$968{\cdot}12\frac{1}{2}$. He sold his wheat for $\$739{\cdot}18\frac{3}{4}$, and his pork for $\$784{\cdot}37\frac{1}{2}$. Did he gain or lose by the operation, and how much?

9. A speculator bought at one time 347·35 acres of land; at another, 637·25 acres; and at another, 1435·7 acres. He is desirous of making his purchases amount to 1225·5 acres. How much land does he still want?

MULTIPLICATION OF DECIMALS AND THE UNITED STATES CURRENCY.

ART. 149. One-tenth taken two times, or multiplied by 2, gives for a product $\frac{2}{10}$; if taken once, or multiplied by 1, the product will be $\frac{1}{10}$; if taken one-tenth of a time, or multiplied by $\frac{1}{10}$ of 1, the product must be $\frac{1}{10}$ of $\frac{1}{10}$ $=\frac{1}{100}$;—thus, $\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}$; $\frac{1}{100}\times\frac{1}{10}=\frac{1}{1000}$; $\frac{1}{1000}\times\frac{1}{10}$ $=\frac{1}{10000}$, &c., which decimally expressed becomes $\cdot1\times\cdot1$ $=\cdot01$; $\cdot01\times\cdot1=\cdot001$; $\cdot001\times\cdot01=\cdot00001$, &c. From which we observe that the number of decimal places in the product is equal to the number of ciphers (which in practice is understood) in the denominators of both factors, which is always equal to the number of decimal places in the two factors. Hence to multiply one decimal by another we proceed as in whole numbers, and *from the right of the product, point off as many places for decimals as there are decimal places in both multiplier and multiplicand.* Should there not be places enough in the product, prefix ciphers.

1. Multiply 4·86 by 7·39.
2. Multiply 14·683 by 10·83.
3. Multiply 122· by 46·7832.
4. What is the product of 202·002 and 7·0002 ?
5. What is the product of 165·3701 and 47·8201 ?
6. What is the product of 3786·478 and 831·0241 ?
7. What is the product of 8602·8312 and 48·76324 ?

ART. **150.** *A decimal is multiplied by* 10, 100, 1000, *&c., by merely removing the decimal point as many places to the right as there are ciphers in the multiplier. If necessary annex ciphers to the number.*

$$\text{Multiply} \left\{ \begin{array}{r} 86{\cdot}723 \\ 14{\cdot}243 \\ 1001{\cdot}001 \end{array} \right\} \text{by } 10.$$

$$\text{Multiply} \left\{ \begin{array}{r} 8{\cdot}076 \\ 41{\cdot}3421 \\ 716{\cdot}311 \end{array} \right\} \text{by } 100.$$

$$\text{Multiply} \left\{ \begin{array}{r} 1{\cdot}832 \\ 30{\cdot}12 \\ 8{\cdot}63412 \end{array} \right\} \text{by } 1000.$$

$$\text{Multiply} \left\{ \begin{array}{r} 148{\cdot}63 \\ 7{\cdot}34876 \\ 28{\cdot}31017 \\ 186{\cdot}4 \\ 2{\cdot}7 \end{array} \right\} \text{by } 10000.$$

PRACTICAL QUESTIONS.

1. What cost 95 tons of hay, at $12·75 a ton ?
2. What cost 125 yards of broadcloth, at $5·37½ a yard ?
3. What cost 275 bushels of potatoes, at $·62½ a bushel ?
4. What cost 384 barrels of sugar, at $17·87½ a barrel ?
5. What cost 312 pounds of butter, at $·18¾ a pound ?
6. What cost 245 barrels of molasses, at $23·18¾ a barrel ?

7. If 25 men earn $\$37{\cdot}18\frac{3}{4}$ in one day, how much can they earn in a year, of 365 days? (not counting Sundays.

8. A gentleman purchased a farm containing 445·5 acres, at $\$34{\cdot}12\frac{1}{2}$ an acre; how much did the farm cost him?

9. How much should be paid for 25·5 cwt. of tobacco, at $\$12{\cdot}37\frac{1}{2}$ a hundred weight?

10. Bought 275 sheep, at $\$1{\cdot}87\frac{1}{2}$ a head, and sold them, at $\$2{\cdot}12\frac{1}{2}$ a head; how much did I gain by the operation?

Division of Decimals and the United States Currency.

ART. **151.** *The quotient arising from dividing any number by another of the same denomination, is a whole number.* Thus, if units be divided by units, tenths by tenths, hundredths by hundredths, or thousandths by thousandths, &c., the quotient will be a whole number. Therefore, in the division of decimals, when the divisor and dividend each contain the same number of decimal places, the quotient will be a whole number; and if the dividend contain more decimal places than the divisor, there must of necessity be as many decimal places in the quotient as the number of decimal places in the dividend exceed the number of decimal places in the divisor.

We deduce the same conclusion from the following considerations.

In the multiplication of decimals, the number of decimal places in the product equals the number of decimal places in both factors. In the division of decimals, the divisor and quotient are multiplied together to produce the dividend; therefore, there must be as many decimal places in the quotient as those in the dividend exceed those in the divisor.

The pupil should bear in mind that he can affix ciphers to the dividend without changing its value; and when necessary, he should prefix ciphers to the quotient.

1. Divide .0016016 by 1.12.

OPERATION.

```
1·12)·0016016(·00143 Ans.
      112
      ---
       481
       448
       ---
        336
        336
        ---
          0
```

EXPLANATION.—As the number of places in the quotient was not equal to the number of decimal places in the dividend minus the number of decimal places in the divisor, so the two ciphers were prefixed that the required number of decimal places could be cut off.

2. Divide ·00144 by 1·2.
3. Divide ·0000075 by ·005.
4. Divide 86·4 by ·24.
5. Divide 59·74514 by 13·6.
6. Divide ·001728 by 4·8.
7. Divide 25·49052 by 24·6.
8. Divide 2448 by ·012

ART. **152.** A decimal may be divided by 10, 100, 1000, &c., by removing the decimal point as many places to the left as there are ciphers in the divisor. If necessary, prefix ciphers to the dividend.

$$\text{Divide} \left\{\begin{matrix} 4{\cdot}36 \\ 80{\cdot}21 \\ 73{\cdot}41 \\ 431{\cdot}2 \end{matrix}\right\} \text{by } 10.$$

$$\text{Divide} \left\{\begin{matrix} 146{\cdot}34 \\ 3{\cdot}24 \\ 36{\cdot}741 \\ 14683 \end{matrix}\right\} \text{by } 100.$$

$$\text{Divide} \left\{\begin{matrix} 47632{\cdot}1 \\ 1478{\cdot}3 \\ 231{\cdot}46 \\ 76{\cdot}041 \\ 31046{\cdot}1 \end{matrix}\right\} \text{by } 1000.$$

PRACTICAL QUESTIONS.

1. If 128 barrels of flour be worth $784, what is the value of 1 barrel?

2. If 54 acres of land cost $816·75, how much is that an acre?

3. What cost 1 yard of broadcloth, if 46 yards cost $263·12?

4. What cost 1 horse, if 34 horses cost $4662·1?

5. What cost 1 bushel of apples, if 70 bushels cost $43·75?

6. If 137 bushels of onions cost $154·12½, what will 1 bushel cost?

7. If 75 quarts of strawberries cost $4·6875, what will 1 quart cost?

8. If 275 bushels of corn cost $171·87½, how much will 1 bushel cost?

PRACTICAL QUESTIONS IN DECIMALS AND THE UNITED STATES CURRENCY.

1. What cost 8640 brick, at $4·25 a 1000?

SOLUTION.—If 1000 brick cost $4·25, 1 brick will cost one thousandth of $4·25, which is $·00425, and 8640 brick will cost 8640 times $·00425, which is $36·72.

2. What will be the cost of 4832 feet of boards, at $6·50 a 1000?

3. What will be the cost of 28460 feet of lumber, at $2·18¾ a hundred?

4. What cost 17640 feet of timber, at $9·45 a 1000?

5. What cost 586 feet of pine boards, at $25·12½ a 1000?

6. What must be paid for planing 46324 feet of boards, at $1·45 a 1000?

7. What is the value of 14672 feet of hemlock boards, at $6·37½ a 1000?

8. A speculator bought 500 acres of land for $987⅘; and 250 acres for $647⅚. He sold 478½ acres for $1245½. How much land has he remaining, and for what must he sell it per acre, so as to neither gain nor lose by the operation?

9. Having deposited in a bank $1860·50; I drew out at one time $84·87½; at another, $47·12½; at another,

$485·18$\frac{3}{4}$; and at another, $147·37$\frac{1}{2}$. How much have I remaining in the bank?

10. A gentleman, who was on a journey of 247$\frac{1}{2}$ miles, traveled 4 days, at the rate of 42$\frac{3}{4}$ miles a day; what distance still remains to be traveled?

11. Bought a house and lot for $3240·50, and paid for improvements on the same $685·87$\frac{1}{3}$. I then sold the property for $4985·62$\frac{1}{2}$. How much did I gain by the transaction?

12. A land dealer has in one farm 195·75 acres; in another 465$\frac{2}{5}$ acres; in another 483$\frac{3}{4}$ acres. He sold 75$\frac{1}{2}$ acres from each. How many acres has he left?

13. A drover bought cattle, for $175·84; mules for $286·95; horses, for $384·87$\frac{1}{2}$; and sold them all for $1847·12$\frac{1}{2}$. How much did he gain by the speculation?

14. A merchant bought cloth for $246·84; silks for $387·87$\frac{1}{2}$; and sugar for $865·18$\frac{3}{4}$. He sold the cloth at a profit of $98·75; the silks, at a loss of $104·12$\frac{1}{2}$; and the sugar, at a profit of $146·18$\frac{3}{4}$. Did he gain or lose, and how much?

15. A merchant bought 47·5 yards of cloth, at $4·75 a yd.; and sold it, at $6·12$\frac{1}{2}$ a yard. How much did he gain?

16. Bought 285 sheep, at $2·12$\frac{1}{2}$ each; and sold them for 25 young cattle. For what must I sell the cattle a head so as to make $75 by the operation?

17. How much money must be paid for 4·5 cwt. of ham, at $14·25 a cwt.; 14 barrels of flour, at $5·37$\frac{1}{2}$ a barrel; 8 barrels of fish, at $9·62$\frac{1}{2}$ a barrel; and 5$\frac{3}{4}$ barrels of sugar, at $19·30 a barrel?

18. What sum of money should be paid for 75·75 pounds of sugar, at $·1125 a pound; 14 lbs. of tea, at $1·37$\frac{1}{2}$ a pound; 15 lbs. of chocolate, at $·125 a pound; and 5·75 gallons of molasses, at $·37$\frac{1}{2}$ a gallon?

19. A speculator bought 147$\frac{1}{2}$ acres of land, at $27·12$\frac{1}{2}$ an acre; and 232$\frac{3}{4}$ acres, at 35\frac{4}{5}$ an acre. He sold the first tract, at $32·18$\frac{3}{4}$ an acre; and the second, at $28·37$\frac{1}{2}$ an acre. Did he gain or lose by the operation, and how much?

20. Bought 4 pieces of cloth, each containing 47$\frac{1}{2}$ yards,

for \$863·25; of which $25\frac{3}{4}$ yards have been sold, at \$6·87$\frac{1}{2}$ a yard. What will be the gain or loss on the whole, if the remainder be sold, at \$5·95 a yard?

21. A drover bought 247 cattle, at \$25·87$\frac{1}{2}$ each. He sold 84 of them, at \$32·75 each; 45 of them, at \$22·45 each; and the remainder, at \$28·12$\frac{1}{2}$ each. How much did he gain by the speculation?

22. A merchant barters to a farmer, 18·75 yards of broadcloth, at \$7·12$\frac{1}{2}$ a yard; $47\frac{1}{2}$ yards of muslin, at \$·09$\frac{3}{4}$ a yard; 6 pair of boots, at \$4·37$\frac{1}{2}$ a pair;—for 47 bushels of corn, at \$·57 a bushel; $65\frac{3}{4}$ bushels of wheat, at \$1·12$\frac{1}{2}$ a bushel. The difference in the value of the articles exchanged, is to be paid in money. Which of them must receive money, and how much?

23. A merchant bought 1246 bushels of wheat, at \$1·37$\frac{1}{2}$ a bushel; of which he sold to one man 463 bushels, at \$1·45 a bushel; to another $384\frac{1}{2}$ bushels, at \$1·87$\frac{1}{2}$ a bushel. At what price per bushel must the remainder be sold so as to gain on the whole, at the rate of \$56 on a 1000 bushels?

24. A person, having \$46·87$\frac{1}{2}$ was desirous of purchasing an equal number of pounds of tea, coffee, and sugar; the tea, at \$1·12$\frac{1}{2}$ a pound; the coffee, \$·62$\frac{1}{2}$; and the sugar, \$·12$\frac{1}{2}$ a pound. How many pounds of each could he buy?

25. Find the amount of a store-bill for $15\frac{3}{4}$ yards of cloth, at \$3·37$\frac{1}{2}$ a yard; $20\frac{1}{2}$ yards of silk, at \$1·18$\frac{3}{4}$ a yard; and 15 skeins of thread, at \$·06$\frac{1}{4}$ a skein.

26. Bought 16 barrels of sugar for \$425·25, and sold the same at a profit of \$1·87$\frac{1}{2}$ a barrel. At what price per barrel was it sold, and what was the entire profit?

27. A merchant bought 35 pieces of broadcloth, each containing $18\frac{3}{4}$ yards, at \$6·18$\frac{3}{4}$ a yard; and sold it so as to clear, after deducting \$4·37$\frac{1}{2}$ for his trouble, \$89·87$\frac{1}{2}$. At what price per yard was the cloth sold?

28. What is the value of sugar a cwt. when ·75 cwt. cost \$6·375; and what should be paid for $16\frac{3}{4}$ cwt. of sugar, at the same rate?

29. A merchant bought of one farmer $225\frac{3}{4}$ bushels of

wheat, and of another $106\frac{1}{2}$ bushels, at $\$1{\cdot}18\frac{3}{4}$ a bushel. He made 195 bushels of it into flour; and sold the flour, at a profit of $\$125{\cdot}87\frac{1}{2}$. Will he gain or lose, if he sells the remainder of the wheat, at $\${\cdot}93\frac{3}{4}$ a bushel.

30. A drover bought 146 cows, at $\$27{\cdot}18\frac{3}{4}$ a head; and 166 sheep, at $\$1{\cdot}87\frac{1}{2}$ each. He sold 83 of the cows, at $\$28{\cdot}12\frac{1}{2}$ a head; and all of the sheep, at $\$1{\cdot}81\frac{1}{4}$ each. At what rate per head must he sell the remainder of his cows so as to make a profit of $\$125{\cdot}93\frac{3}{4}$ on the whole?

ART. 153. REDUCTION OF DENOMINATE NUMBERS TO DECIMALS.

1. Reduce 15*s.* 9*d.* 3 far. to the decimal of a pound.

OPERATION.

	far.
4	3·00
12	9·75 *d.*
20	15·8125 *s.*
	·790625 of a pound.

EXPLANATION.—We annex two ciphers to the 3 far., which reduces it to hundredths. 4 far. make 1 penny; therefore, $\frac{1}{4}$ of the number of farthings will equal the number of pence, which is ·75*d.* This being annexed to the 9*d.* = 9·75*d.* We next divide this by 12, to reduce it to the decimal of a shilling. and obtain ·8125*s.*; which, being annexed to the 15*s.* gives 15·8125*s.* We now divide this by 20, to reduce it to the decimal of a pound, and obtain ·790625 of a pound for the answer.

2. Reduce 18*s.* 9*d.* 2 far. to the decimal of a pound. sterling.

3. Reduce 1 ft. 8 inches, to the decimal of a yard.

4. Reduce 1002·15 pwt. 9 grs., to the decimal of a pound Troy.

5. Reduce 15 cwt. 3 qrs. 15·45 lbs., to the decimal of a ton.

6. Reduce 5 fur. 25 rds., to the decimal of a mile.

7. Reduce 2 R. 25·5 P., to the decimal of an acre.

8. Reduce 6 fur. 15 rds. 3 yds. 2 ft. 10 in., to the decimal of a mile.

9. Reduce £4 15*s.* 10*d.* 1 farthings, to the decimal of a pound.

10. Reduce 6 T. 12 cwt. 2 qrs. 14 lbs. 10 oz. 8 dr. to the decimal of a ton.

ART. 154. REDUCTION OF DENOMINATE DECIMALS, TO WHOLE NUMBERS OF A LOWER DENOMINATIONS.

1. Reduce ·735 of a pound, to shillings, pence and farthings.

OPERATION.

$$\begin{array}{r l} \pounds\ \cdot735 & \\ 20 & \\ \hline 14{\cdot}700 & s. \\ 12 & \\ \hline 8{\cdot}400 & d. \\ 4 & \\ \hline 1{\cdot}600 & \text{far.} \end{array}$$

EXPLANATION.—I wish to reduce ·735 of a pound to shillings. There are 20*s.* in £1; therefore, 20 times the number of pounds = the number of shillings, 20 × ·735 = 14·7*s.* In ·7*s.*, how many pence? There are 12*d.* in 1*s.*; therefore, 12 times the number of shillings equal the number of pence. 12 × ·7 = 8·4*d.* In ·4*d.* how many farthings? There are 4 farthings in 1 penny; therefore, 4 times thenumber of pence equal the number of farthings 4 × ·4*d.* = 1·6 far. Therefore, £0·735 = 14*s.* 8*d.* 1·6 far.

2. What is the value of ·389 of a pound sterling?
3. What is the value of ·635 of a yard?
4. What is the value of ·45½ of an ell French?
5. What is the value of ·832 of an ell English?
6. What is the value of ·75¾ of a mile?
7. What is the value of ·895 of an acre?
8. What is the value of ·975625 of a pound Troy?
9. What is the value of ·875 of a score?
10. What is the value of ·95625 of a ream of paper?
11. What is the value of $\cdot85\frac{1}{7}$ of a firkin of butter?
12. What is the value of ·7575 of a great gross?
13. What is the value of $\cdot\dot{1}2\dot{3}$ of a pound sterling?
14. What is the value of $\cdot\dot{1}4285\dot{7}$ of a bushel of salt?
15. What is the value of $\cdot78\dot{3}$ of a bushel of wheat?
16. What is the value of $\cdot857\dot{1}4285\dot{7}$ of a bushel of corn or rye?
17. What is the value of $\cdot08\dot{3}$ of a pound sterling?
18. What is the value of $\cdot1\dot{6}$ of a cwt.?

19. What is the value of $\cdot 12\dot{3}$ of a mile?

20. What is the value of $\cdot 46\dot{3}$ of a ton?

PRACTICAL QUESTIONS.

1. What is the value of 3 cwt. 2 qrs. 15 lbs. of sugar at \$5·75 a cwt.?

2. What is the value of 15 gallons, 3 qt. 1 pt. of molasses, at \$·87½ a gallon?

3. What is the value of 16 bushels, 2 pks. 7 qts. of rye at \$1·37½ a bushel?

4. What is the value of 84 yds. 3 qrs. 3 nas. of broadcloth, at \$5·87½ a yard?

5. What is the value of 16 cwt. 2 qrs. 14·5 lbs. of pork, at \$14·93¾ a cwt.?

6. What is the value of 84 T. 14 cwt. 2 qrs. 15 lbs. of hay, at \$14·18¾ a ton?

7. What is the value of 34 lbs. 8½ oz. of butter, at \$·18¾ a pound?

8. What will it cost to construct 14 miles, 5 fur. 25 rds. of plank road, at \$1437·62½ per mile?

9. A farmer sold 34 bush. 3 pks. 7 qts. of clover-seed, at \$6·84½ a bushel, and in payment received 40 bushels 2 pks. 1 pt. of grass-seed, at \$3·87½ a bushel. How much remains due?

10. A tailor paid \$1468·75 for 385 yds. 3 qrs. 3 nas. of cloth; ⅔ of which he sold, at \$4·37½ a yard; and the remainder, at \$5·93¾ a yard. How much did he gain by the bargain?

11. If ¾ of a ton of hay cost \$8·87½, what will 4 T. 15 cwt. 3 qrs. cost?

12. A merchant bought 125 hhds. 30·5 gals. 3 qts. of molasses for \$1585·12½; and sold ⅗ of it for \$21·75 a hogshead; and the remainder, at \$28·93¾ a hogshead. How much did he gain by the operation?

Reduction of Denominate Fractions.

ART. **155.** A *Denominate fraction* is a fraction of any denominate number; as ⅖ of a yard, ⅔ of a mile, &c.

Reduction of denominate fractions is changing them from one denomination to another without altering their value.

1. Reduce $\frac{9}{896}$ of a gallon to the fraction of a gill.

OPERATION BY CANCELLATION.

$$\overset{\text{gal.}}{\frac{9}{\cancel{896}}} \times \frac{\cancel{4}}{1} \times \frac{\cancel{2}}{1} \times \frac{\cancel{4}}{1} = \frac{9}{28} \text{ of a gill.}$$

$$\begin{array}{l} \cancel{224} \\ \cancel{112} \\ 28 \end{array}$$

EXPLANATION.—There are 4 quarts in 1 gallon; therefore, 4 times the number of gallons equal the number of quarts. $\frac{9}{896} \times 4 = \frac{9}{224}$ of a quart; (which, for convenience, may be read in the form of a compound fraction. There are 2 pints in 1 quart; therefore, twice the number of quarts equal the number of pints. $\frac{9}{896} \times \frac{4}{1} \times \frac{2}{1} = \frac{9}{112}$ of a pint. There are 4 gills in 1 pint; therefore, 4 times the number of pints equal the number of gills. $\frac{9}{896} \times \frac{4}{1} \times \frac{2}{1} \times \frac{4}{1}$, equals the number of gills, which, when cancelled, becomes $\frac{9}{28}$ of a gill.)

2. Reduce $\frac{1}{640}$ of a pound to the fraction of a farthing.
3. What part of a grain is $\frac{1}{28800}$ of a pound Troy?
4. What part of a pint is $\frac{7}{4480}$ of a bushel?
5. What part of a pound is $\frac{7}{14000}$ of a ton?
6. What part of a second is $\frac{3}{103680}$ of a day?
7. What part of a foot is $\frac{2}{1980}$ of a furlong?
8. What part of a dram is $\frac{1}{20480}$ of a hundredweight?

ART. **156.** REDUCTION OF FRACTIONS OF A LOWER, TO THOSE OF A HIGHER DENOMINATION.

1. Reduce $\frac{6}{7}$ of a farthing to the fraction of a pound.

OPERATION BY CANCELLATION.

$$\overset{\text{far.}}{\frac{\cancel{6}}{7}} \times \frac{1}{4} \times \frac{1}{\underset{2}{\cancel{12}}} \times \frac{1}{20} = \frac{1}{1120} \text{ of a pound.}$$

EXPLANATION.—$\frac{6}{7}$ of a farthing is what part of a penny? 4 farthings make 1 penny; therefore, $\frac{1}{4}$ of the number of farthings equals the number of pence. By a similar method of

reasoning we find $\frac{1}{12}$ of the number of pence equal the number of shillings; and $\frac{1}{20}$ of the number of shillings equal the number of pounds.

2. What part of a pound Troy, is $\frac{4}{5}$ of a grain?
3. What part of an acre is $1\frac{4}{5}$ feet?
4. What part of 10 days is $\frac{3}{4}$ of a minute?
5. What part of 20 bushels is $\frac{2}{3}$ of $\frac{3}{4}$ of a gill?
6. What part of a rod is $\frac{7}{12}$ of $2\frac{3}{4}$ inches?
7. What part of 8 miles is $\frac{5}{6}$ of a rod?
8. What part of a yard is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of an ell French?

ART. **157.** REDUCTION OF SIMPLE OR DENOMINATE NUMBERS, TO THE FRACTIONAL PART OF ANOTHER SIMPLE OR DENOMINATE NUMBER.

1. What part of £1 is 10*s.* 6*d.* 1 far.?

OPERATION.

$$\frac{10s.\ 6d.\ 1\text{ far.} = 505\text{ far.}}{= 960\text{ far.}} = \frac{101}{192}\text{ part.}$$

SOLUTION.—4 farthings make 1 penny; therefore 1 farthing is $\frac{1}{4}$ of a penny. $6\frac{1}{4}d. = \frac{25}{4}d.$ 12*d.* make 1 shilling; therefore $\frac{1}{12}$ of the number of pence equals the number of shillings. $\frac{1}{12} \times \frac{25}{4} = \frac{25}{48}s.$ $10\frac{25}{48}s. = \frac{505}{48}s.$ 20*s.* make £1; therefore $\frac{1}{20}$ of the number of shillings equals the number of £. $\frac{1}{20} \times \frac{505}{48} = \frac{101}{192}$£.

2. What part of 3 yds. is 4 E. Fr. 2 qrs.?
3. What part of 3 cwt. 3 qrs. is 2 cwt. 3 qrs. 15 lbs.?
4. What part of 3 A. 3 R. $32\frac{1}{3}$ P. is 2 A. 2 R. 30 P.?
5. What part of 3 feet square is 3 square feet?

ART. **158.** TO FIND THE VALUE OF A DENOMINATE FRACTION, IN WHOLE NUMBERS, OF A LOWER DENOMINATION.

1. What is the value of $\frac{5}{7}$ of a pound sterling?

OPERATION

$$\begin{array}{r|cccc} & & 20 & 12 & 4 \\ & £ & s. & d. & qr. \\ 7) & 5 & 0 & 0 & 0 \\ \hline & 0 & 14 & 3 & 1\frac{5}{7} \end{array} \text{ Ans.}$$

EXPLANATION.—I wish to find $\frac{5}{7}$ of £1, but $\frac{5}{7}$ of £1 is the same as $\frac{1}{7}$ of £5; hence, I find $\frac{1}{7}$ of £5.

2. What is the value of $\frac{4}{5}$ of a shilling?
3. What is the value of $\frac{5}{9}$ of a cwt.?
4. What is the value of $\frac{6}{8}$ of a yard?
5. What is the value of $\frac{27}{75}$ of a day?
6. What is the value of $\frac{7}{8}$ of a mile?
7. What is the value of $\frac{21}{24}$ of a hogshead of wine?
8. What is the value of $\frac{7}{8}$ of a year?
9. What is the value of $\frac{3}{7}$ of an ell French?
10. What is the value of $\frac{9}{10}$ of a ton?

ADDITION OF DENOMINATE FRACTIONS.

ART. **159.** We have learned that whole numbers of different denominations connot be added; the same is true of fractions of different denominations. Hence, we first find the value of the given fractions by Art. 158; then add them together.

1. Add $\frac{11}{15}$ of a pound to $\frac{6}{7}$ of a shilling.

OPERATION.

$\frac{11}{15}$ of a pound $= 14s.\ 8d.$
$\frac{6}{7}$ of a shilling. $= 10d.\ 1\frac{1}{7}$ far.

Ans. $15s.\ 6d.\ 1\frac{1}{7}$ far.

2. Add $\frac{2}{3}$ of a pound to $\frac{8}{16}$ of a shilling.
3. Add $\frac{9}{10}$ of a cwt. to $\frac{4}{5}$ of a quarter.
4. Add $\frac{5}{7}$ of a ton to $\frac{16}{17}$ of a cwt.
5. Add $\frac{8}{9}$ of a mile to $\frac{4}{5}$ of a furlong.
6. Add $\frac{4}{5}$ of an acre to $\frac{7}{8}$ of a rood.
7. Add $\frac{8}{9}$ of a hogshead to $\frac{7}{8}$ of a gallon.
8. Add together $\frac{6}{7}$ of a bush., $\frac{2}{3}$ of a peck, and $\frac{4}{5}$ of a quarter.
9. Add together $\frac{4}{7}$ of a ton, $\frac{4}{5}$ of a cwt., and $\frac{5}{7}$ of a qr.
10. Add together $\frac{2}{3}$ of a month, $\frac{4}{5}$ of a week, and $\frac{6}{7}$ of a day.

Art. **160.** Subtraction of Denominate Fractions.

1. From $\frac{7}{9}$ of a mile subtract $\frac{5}{7}$ of a furlong.

OPERATION.

	40	$5\frac{1}{2}$	3	12	
	fur.	rds.	yds.	ft.	in.
$\frac{7}{9}$ of a mile =	6	8	4	2	8
$\frac{5}{7}$ of a fur. =		28	3	0	$5\frac{1}{7}$
Ans.	5	20	1	2	$2\frac{6}{7}$

2. From $\frac{7}{8}$ of a bushel take $\frac{13}{16}$ of a peck.
3. From $\frac{6}{7}$ of a week take $\frac{2}{3}$ of a day.
4. From $\frac{5}{7}$ of 25 yards take $\frac{3}{4}$ of 6 E. French.
5. From $\frac{11}{12}$ of 23 tons take $\frac{6}{7}$ of 18 cwt.
6. A company agree to construct 25 miles, 8 fur. 18 rds. of road, but after constructing 6 mi. 2 fur. 23 rds. and 2 ft. more than $\frac{2}{5}$ of it, they relinquish the job. How much remains to be constructed?
7. A merchant bought $\frac{8}{9}$ of 15 hhd. 42 gals. of molasses, and sold $\frac{4}{5}$ of 2 hhd. 53 gals. of it. How much remained unsold?
8. A merchant bought 14 cwt. 3 qrs. 18 lbs. of sugar, and sold $\frac{2}{3}$ of it, lacking 4 cwt. 1 qr. 15 lbs.; how much remains unsold?

PRACTICAL QUESTIONS.

1. What is the value of $\frac{5}{7}$ of 15 yards of cloth, at \$5·62½ a yard?
2. What is the value of $\frac{5}{8}$ of 3 bushels, 3 pks. 7 qts. of gooseberries, at \$·06¼ a quart?
3. What cost $\frac{8}{9}$ of 47 cords, 110 feet of wood, at \$5·87½ a cord?
4. What cost 1 pound of tea, if $11\frac{1}{3}$ pounds cost \$13·826?
5. What will 6 cwt. 3 qrs. 20 lbs. of honey cost, at \$18·87½ a cwt.?
6. What will 14 bushels, 2 pks. 7 qts. 1 pt. of grass-seed cost, at \$6·62½ a bushel?
7. If it require 4 hours 20 minutes for a man to cut

1 cord of wood, how many days of 8 hours and 40 minutes each, will be required to cut 847 cords 84 feet?

8. Four persons share 625 pounds of sugar as follows: the first takes $\frac{1}{8}$ of $\frac{4}{5}$ of the whole; the second takes $\frac{1}{7}$ of $\frac{7}{8}$ of the remainder; the third takes $\frac{7}{8}$ of $\frac{32}{35}$ of what now remains; and the fourth takes what is left. How much did each receive?

9. A received $\frac{1}{8}$ of a certain quantity of molasses; B $\frac{1}{9}$; C $\frac{3}{5}$ of the remainder; and D what then remained. It now appears that C has 64 gals. more than A and B together. How much did each receive?

10. A farmer, owning 864 A. 3 R. 39 P. of land, divided $\frac{6}{7}$ of it equally among 4 of his sons. How much did each son receive, and how many acres had the father remaining?

11. Bought 184 gals. 3 qts. of molasses, at \$·37$\frac{1}{2}$ a gallon, and used 27 gals. 2 qts. of it; how must I sell the remainder per gallon so as to receive \$3·84$\frac{1}{2}$ more than the whole cost?

12. A person gave $\frac{1}{4}$ of all his money for a horse; $\frac{1}{2}$ of the remainder for a colt; and $\frac{2}{3}$ of what then remained for a cow. He then had remaining \$8·87$\frac{1}{2}$. What was the cost of each, and how much money had he at first?

13. A merchant gave for some raisins $\frac{1}{6}$ of all his money; for some cinnamon $\frac{1}{9}$ of all his money; for some sugar $\frac{1}{2}$ of what remained; for some flour $\frac{2}{3}$ of what then remained; and what still remained he gave for some butter. What did each article cost him, providing the sugar cost \$136·18$\frac{3}{4}$ more than the flour?

14. A certain sum of money is to be divided among 4 persons; the first is to have $\frac{1}{5}$ of it; the second $\frac{1}{3}$ of it; the third $\frac{2}{3}$ of what remains; and the fourth the remainder. What was the sum to be divided, and how much did each receive, providing the third received \$147·93$\frac{3}{4}$ less than the first and second together?

15. How much butter at \$·18$\frac{3}{4}$ a pound, must be given for 25 gals. 3 qts. 1 pt. of molasses, at \$·37$\frac{1}{2}$ a gallon?

16. From a piece of cloth containing 147 yds. 4 E. French, three suits of clothes, each requiring 6 E. English,

were taken. How much would the remainder come to, at $5·18$\frac{3}{4}$ a yard?

17. How many inches in $\frac{5}{6}$ of an E. E.; $\frac{3}{4}$ of an E. Fr.; and $\frac{4}{5}$ of a quarter?

18. A merchant lost from a hogshead of molasses $\frac{1}{8}$ of it, + $\frac{4}{5}$ of a gallon and $\frac{5}{6}$ of a quart. How much of the hogshead, expressed decimally, leaked out, and how much remained in?

19. Bought 15 tons 14 cwt. 3 qrs. 24 lbs. of iron, and sold 10 tons 5 cwt. 1 qr. 15 lbs. of it. What is the value of $\frac{7}{9}$ of what remains, at $·06$\frac{1}{4}$ a pound?

20. Bought a quantity of grain for $358·84; and sold $\frac{13}{17}$ of it to one man; $\frac{5}{9}$ of the remainder to another man; and used $\frac{2}{3}$ of the remainder myself. What is the value of the remainder?

21. A, B, C, and D worked together on this condition: A was to receive $60·06 of it, and $\frac{1}{10}$ of the remainder; B was to receive $70·07 and $\frac{1}{10}$ of the remainder; C was to receive $80·08 and $\frac{1}{10}$ of the remainder; and D took what then remained. By this division each man received the same sum. How much did their wages amount to?

DUODECIMALS.

ART. **161.** DUODECIMALS are a kind of denominate numbers, the denominations of which increase uniformly in a *twelve-fold ratio.* Its denominations are the *foot* (ft.), which is the unit; the inch, or *prime* ('), $\frac{1}{12}$ of the foot, the *second* (''), $\frac{1}{12}$ of the prime; the *third* ('''), $\frac{1}{12}$ of the second; and so on, indefinitely. The accents that distinguish the denominations below feet, are called *Indices.*

Duodecimals are applied to the measurement of *surfaces* and *solids.*

TABLE.

12 Fourths ('''')	make	1 Third,	marked '''
12 Thirds	"	1 Second,	" ''
12 Seconds	"	1 Prime, or Inch,	" '
12 Primes, or Inches	"	1 Foot,	" ft

ADDITION AND SUBTRACTION OF DUODECIMALS.

ART. **162.** Duodecimals are *added* and *subtracted* the same as other Denominate numbers.

1. Add together 6 ft. 4′ 5″ 8‴, 8 ft. 4′ 8″ 9‴, 7 ft. 3′ 8″ 9‴, and 12 ft. 9′ 11″ 10‴.

2. What is the sum of 17 ft. 8′ 9″ 11‴, 14 ft. 6′ 7″, 8 ft. 9′ 11″ 4‴, and 16 ft. 9′ 10″ 11‴ ?

3. What is the sum of 20 ft. 9′ 11″ 6‴ 1⁗, 14 ft. 8′ 9″ 10‴, 12 ft. 9′ 8″ 10‴ 8⁗, 8 ft. 11⁗, and 6 ft. 9′ ?

4. From 84 ft. 8′ 9″ 11‴ 3⁗, subtract 66 ft. 11′ 8′ 4‴ 9⁗.

5. What is the sum, and what is the difference of 84 ft. 3′ 8″ 9‴ 2⁗, and 48 ft. 9′ 7″ 11‴ 10⁗.

6. What is the sum, and what is the difference of 137 ft. 3′ 9″ 4‴ 6⁗ and 98 ft. 9′ 10″ 11‴ 7⁗.

MULTIPLICATIÓN OF DUODECIMALS.

In Duodecimals, the foot, when used to express *surfaces*, contains 144 sq. in., and when used to express *solids*, 1728 cu. in. Consequently, in the measurement of surfaces, 5′ would equal $\frac{5}{12}$ of a square foot, instead of a linear foot; that is, $\frac{5}{12} \times 144$ sq. in. $=60$ sq. in. In the measurement of solids, 5′ would equal $\frac{5}{12}$ of 1728 cu. in. (a cubic foot) $=720$ cu. in.

From the preceding remark we infer that a strip of surface 1 inch wide and 12 inches long, makes 1′ square measure; and that a slab 1 inch thick, 12 inches long, and 12 inches wide, makes 1′ solid measure.

1. What is the product of 8 ft. 5′ by 9 ft. 7′ ?

OPERATION.

8 ft.	5′	
9 ft.	7′	
4 ft.	10′	11″
75 ft.	9′	
80 ft.	7′	11″

EXPLANATION.—$5 = \frac{5}{12}$, and $7' = \frac{7}{12}$ of a foot. Therefore, we say, $7' \times 5' = \frac{35}{144}$ of a foot, which is $35'' = 2'\ 11''$; we write down the 11″ and carry the 2′ to the next product. $7' \times 8$ ft. $= \frac{56}{12}$ of a foot, which is 56′, and 2′ added $=$ 58′, which equals 4 ft. 10′, this we write down. 9 ft. $\times$ 5′ $= \frac{45}{12}$ of a foot, which is $45' = 3$ ft. 9′; write down the 9′ and carry the 3 ft. to the

next product. 9 ft. $\times$ 8 ft. = 72 ft. and 3 ft. added = 75 ft. The sum of these partial products gives the required product, which is 80 ft. 7′ 11″.

REMARK.—It has already been stated that it was impossible to multiply one concrete number by another. The above example may appear at first thought to be contrary to that statement, but it must be remembered that the multiplier is considered an abstract number.

2. What is the product of 14 ft. 7′ 2″ by 6 ft. 3′ 5″ ?

3. What is the area of a marble slab, the length of which is 9 ft. 8′ 11″, and width 3 ft. 7′ ?

4. How many square feet are contained in the floor of a room 40 ft. 10′ long, 32 ft. 8′ wide ?

5. How many square feet in 10 boards, each 18 ft. 10 long and 1 ft. 8′ wide ?

6. How many square feet of boards will it take to inclose a piece of land 80 ft. 10 in. long, and 60 ft. 8 in. wide, with a close fence 7 ft. 6 in. high ?

7. How many square yards in a floor which is 48 ft. 6′ long, and 36 ft. 10′ wide ?

8. What will the plastering of a room cost, at 18 cents a square yard, the length of which is 30 ft. 10 in., width 24 ft. 6 in., and height of ceiling 8 ft. 4′ ?

9. In a certain building there are 32 windows; in each window 16 lights; and each light is 1 ft. 10′ by 11′. How many square feet of glass, in the 32 windows ?

10. In a certain room 24 ft. long, 18 ft. 6′ wide, and 10 ft. 2′ high, there are 6 windows, each 6 ft. 2′ long, 3 ft. 10′ wide; and 3 doors, each 6 ft. 10′ by 3 ft. What will be the cost of plastering this room, at 16 cents a square yard ?

11. How many solid feet in a pile of wood 24 ft. 6 in. long, 6 ft. 5′ high, and 4 ft. 6′ wide ?

REMARK.—Multiply the length, height, and width together, to find the solid contents.

12. How many cubic feet in a stick of timber 32 ft. 9′ long, 2 ft. 2′ wide, and 2 ft. 8′ thick ?

13. How many bricks, each 8 in. long, 4 in. wide, and 2 in. thick, are required to build a wall 144 feet long, 6 ft. 6 in. high, and three bricks wide, no allowance being made for the mortar ?

ART. 164. DIVISION OF DUODECIMALS.

1. There are 8 ft. 5′ 3″ in the surface of a marble slab, the length of which is 3 ft. 9′; what is its width?

OPERATION.

```
3 ft. 9′)8 ft. 5′ 3″(2 ft. 3′ Ans.
         7 ft. 6′
         ------
             11′ 3″
             11′ 3″
         ------
                 0
```

EXPLANATION.—3 ft. is contained in 8 ft. 2 times. Multiplying the whole divisor by 2 ft. give 7 ft. 6′ for the product, which we subtract from the corresponding denominations of the dividend, and obtain 11′ for a remainder, to which annex the next denomination of the dividend, and we have 11′ 3″. 3 ft. is contained in 11′, 3′ times. The divisor being multiplied by this 3′ give 11′ 3″, which being subtracted from the last remainder leaves nothing. Therefore, the marble slab was 2 ft. 3′ in width.

REMARK.—If the student will bear in mind that the superficial contents of any surface is found by multiplying the length by the breadth, he will readily understand that dividing the superficial contents of any surface by its length will give its width, or by its width will give its length. Also, since the solidity of a body is found by multiplying its three dimensions together, if we divide its cubical contents by the product of either two of its dimensions, the quotient will be the other dimension.

The number of indices to be annexed to any term of the quotient can be readily determined. since the *indices of the quotient added to the indices of the divisor must equal those of the dividend.*

2. There are 489 sq. ft. 8′ 0″ 2‴ 7⁗, in the surface of a floor. The length of the floor is 87 ft. 1′ 11″. What is its width?

3. There are 28 sq. ft. 3′ 11″ 2‴, in the surface of a table; the length of which is 6 ft. 9′ 7″; what is its width?

4. The area of a certain pond, the length of which is 43 ft. 9′ 6″, is 1075 sq. ft. 0′ 3″ 0‴ 6⁗. What is its width?

5. A stick of timber is 3 ft. 2′ wide, 2 ft. 11′ thick, and contains 135 cu. ft. 10′ 2″ 1‴. What is its length?

6. The area of a pond is 3978 ft. 1′ 6″; its length is 100 ft. 6′. What is its width?

7. The area of a marble slab is 27 ft. 0′ 7″ 9‴ 6⁗; its length is 7 ft. 6′ 3″. What is its width?

8. The area of a hall is 103 ft. 4′ 5″ 8‴ 4⁗; its width is 6 ft. 11′ 8″. What is its length?

REDUCTION OF CURRENCIES.

ART. **165.** REDUCTION OF CURRENCIES teaches how to find the value of the denominations of one currency in the denominations of another.

The value of a dollar, expressed in shillings and pence, is not the same in different States of the Union, and in different countries. This difference may be learned from the following

TABLE.

\$1 in	New York, Ohio, North Carolina,	$= 8s. = £\frac{2}{5}$, called New York currency.
\$1 in	New England States, Virginia, Kentucky, Tennessee,	$= 6s. = £\frac{3}{10}$, called New England currency.
\$1 in	New Jersey, Pennsylvania, Delaware, Maryland,	$= 7s.\ 6d. = £\frac{3}{8}$, called Pennsylvania currency.
\$1 in	South Carolina, Georgia,	$4s.\ 8d. = £\frac{7}{30}$, called Georgia currency.
\$1 in	Canada, Nova Scotia,	$5s. = £\frac{1}{4}$, called Canada currency.

The legal value of £1 English or Sterling money, is \$4.84, as fixed by an act of Congress in 1842.

The above Table gives the value of \$1, expressed in the fraction of a pound, in the different currencies. The value of £1 in each of the above currencies is found by analysis, thus,—If $£\frac{2}{5} = \$1$, $£\frac{1}{5} = \$\frac{1}{2}$ and $£\frac{5}{5}$ or $£1 = 5$ times $\frac{1}{2}$, which is $\$\frac{5}{2}$. In a similar manner from the above table, we can form the following

TABLE.

$£1 = \$\frac{5}{2}$, New York currency.
$£1 = \$\frac{10}{3}$, New England currency.
$£1 = \$\frac{8}{3}$, Pennsylvania currency.
$£1 = \$\frac{30}{7}$, Georgia currency.
$£1 = \$4$, Canada currency.

1. Reduce $321·75 to its equivalent value in Pennsylvania currency.

OPERATION.

$$\$321{\cdot}75 \times \tfrac{3}{8} = £120{\cdot}65625,$$
which equals £120 13*s.* 1*d.* 2 far.

2. Reduce $345·25 to its equivalent value in New York currency.
3. Reduce $684·12½ to its equivalent value in New England currency.
4. Reduce $67·84 to its equivalent value in Georgia currency.
5. Reduce $846·87½ to its equivalent value, Canada currency.
6. Reduce $846·625 to its equivalent value in English or Sterling money.

ART. **166.** REDUCTION OF POUNDS, SHILLINGS, &C., OF DIFFERENT CURRENCIES, TO FEDERAL MONEY.

1. Reduce £75 15*s.* 6*d.* New York currency, to Federal money.

OPERATION.

$$£75\ 15s.\ 6d. = £75{\cdot}775.$$
$$£75{\cdot}775 \times \tfrac{5}{2} = \$189{\cdot}4375.$$

2. Reduce £154 10*s.* 8*d.* New England currency, to Federal money.
3. Reduce £346 16*s.* 9*d.* Pennsylvania currency, to Federal money.
4. Reduce £843 15*s.* 8*d.* Georgia currency, to Federal money.
5. Reduce £49 18*s.* 11*d.* Canada currency, to Federal money.
6. Reduce £784 17*s.* 10*d.* Sterling, to Federal money.

ART. **167.** The following table shows the value of some of the foreign coins at their standard value :—

1 Pound Sterling, or Sovereign, . . .	$4·84
1 Guinea, English, ...	5·00
1 Crown, ...	1·06

1 Shilling piece, English,	·23
1 Franc,	·186
1 Doubloon, Mexico,	15·60
1 Specie Dollar of Sweden and Norway,	1·06
1 Specie Dollar of Denmark,	1·05
1 Thaler of Prussia and N. States of Germany,	·69
1 Florin of Austrian Empire and City of Augsburg,	·485
1 Ducat of Naples,	·80
1 Ounce of Sicily,	2·40
1 Pound of British Provinces, Nova Scotia, New Brunswick, Newfoundland, and Canada,	4·00

Note.—A little reflection will enable the pupil to reduce any of these foreign coins to Federal Money, or Federal Money to foreign coins.

Aliquot Parts.

Art. **168.** The *half*, *third*, *fourth*, *fifth*, &c., of any quantity, is an *Aliquot Part* of that quantity.

Art. **169.** Analysis is applied to arithmetical solutions, when the various factors of the question and their relations are traced out, forming a process of reasoning.

ANALYSIS BY ALIQUOT PARTS.

1. What is the value of 4 cwt. 2 qrs. $12\frac{1}{2}$ lbs. of sugar, at $8.84 a cwt ?

OPERATION.

2 qrs. = $\frac{1}{2}$ cwt.	$ 8.84
	4
	35.36 value of 4 cwt.
	4.42 value of 2 qrs.
$12\frac{1}{2}$ lbs. = $\frac{1}{4}$ of 2 qrs.	$1.10\frac{1}{2}$ value of $12\frac{1}{2}$ lbs.
	$$40.88\frac{1}{2}$ Ans.

2. What is the value of 25 lbs. 5 oz 12 pwts. of silver ware, at $$54·18\frac{3}{4}$ a pound ?

3. What is the value of 6 tons 5 cwt 3 qrs. of iron, at $$35·37\frac{1}{2}$ a ton ?

4. What is the value of 16 cwt. 2 qrs 15 lbs. of sugar at $$9·37\frac{1}{2}$ a cwt. ?

5. What is the value of 346 bushels 3 pks. 1 qt. of rye, at $\$\cdot93\frac{3}{4}$ a bushel?

6. What is the value of 3 pks. 6 qts. of cherries, at $\$1\cdot12\frac{1}{2}$ a peck?

7. A market woman bought 2 bushels 3 pks. 4 qts. of strawberries, at \$2·87 a bushel. How much did she pay for them?

8. A merchant bought 25 yds. 2 qrs. 2 nas. of silk, at $\$1\cdot87\frac{1}{2}$ a yard; and 37 yds. 3 qrs. 3 nas. of broadcloth, at \$4·95 a yard. What did the whole amount to?

9. A gentleman bought a lot of land containing 47 A. 2 R. 25 P., at $\$85\cdot37\frac{1}{2}$ an acre. How much did he pay for the lot?

CANCELLATION.

ART. **170.** *Cancellation*, in arithmetic, consists in *rejecting equal factors* from a divisor and dividend, which does not change the value of the fraction; it being the same as dividing both divisor and dividend by the same number. (See Art. 116. Proposition 6th.)

ANALYSIS BY CANCELLATION.

1. If $\frac{2}{3}$ of a yd. of cloth cost $\$\frac{8}{9}$, what will $\frac{5}{8}$ of a yd. cost?

ANALYSIS.—If $\frac{2}{3}$ of a yard cost $\$\frac{8}{9}$, $\frac{1}{3}$ of a yard will cost $\frac{1}{2}$ of $\$\frac{8}{9}$; and $\frac{3}{3}$ (1 yard) will cost $\frac{3}{2}$ of $\$\frac{8}{9}$. If 1 yard cost $\frac{3}{2}$ of $\$\frac{8}{9}$, $\frac{1}{8}$ of a yard will cost $\frac{1}{8}$ of $\frac{3}{2}$ of $\$\frac{8}{9}$; and $\frac{5}{8}$ of a yard will cost $\frac{5}{8}$ of $\frac{3}{2}$ of $\$\frac{8}{9} = \$\frac{5}{6}$. Ans.

OPERATION.

$$\frac{5}{\cancel{8}}\times\frac{\cancel{3}}{2}\times\ \$\frac{\cancel{8}}{\underset{3}{\cancel{9}}}=\$\ \frac{5}{6}\ \text{Ans.}$$

REMARK.—Those who prefer can place the numerators of the fractions on the right of a perpendicular line, one under another; and the denominators, in a similar way, on the left of the same line, and thereby avoid writing the sign of multiplication. Thus:

$$\begin{array}{r|l} \cancel{8} & 5 \\ 2 & \cancel{3} \\ 3\ \cancel{9} & \cancel{8}\ \text{Ans. } \$\frac{5}{6}. \end{array}$$

2. If $\frac{2}{3}$ of a yard of cloth cost \$6, what will $\frac{5}{6}$ of a yard cost?

3. How much will $\frac{5}{8}$ of a ton of hay cost, when $4\frac{1}{3}$ tons cost \$13·39?

4. Allowing a horse to travel $\frac{7}{8}$ of a mile in 4 minutes, what distance would he travel in 48 minutes?

5. If 6 men can perform a certain piece of work in 24·6 days, in what time can 24 men perform the same work?

6. A gave towards the building of a church \$140, which was $\frac{5}{7}$ as much as B gave, and B gave $\frac{3}{7}$ as much as C. How much did C give?

7. If $3\frac{1}{3}$ bushels of corn are worth $2\frac{1}{2}$ bushels of rye, how many bushels of corn are worth $14\frac{2}{3}$ bushels of rye?

8. A has $\frac{4}{5}$ as much money as B; and $\frac{5}{6}$ as much as C, who has $\frac{6}{7}$ as much as D, who has \$2400. How much have A, B, and C respectively?

CHAPTER VII.

RATIO.

ART. **171.** Two numbers or quantities of the same denomination, may be compared together in two ways,—

First. By means of an *Arithmetical Ratio*, which is expressed by their *difference*.

Secondly. By means of a *Geometrical Ratio*, which is expressed by the *number* of *times* the one contains the other.

The word *Ratio*, when used alone, refers to a geometrical ratio.

RATIO is the relation which one number, or quantity, bears to another of the same denomination, and is expressed by the quotient arising from dividing the *first* by the *second*, or by dividing the *second* by the *first*.

When we speak of the ratio of one number to another in Arithmetic, we shall refer to the quotient arising from dividing the *second* term by the FIRST, as the *first* term in

a simple proportion is made the divisor. Thus, the ratio of 2 feet to 8 feet is 4, or expressed in the form of a fraction, is $\frac{8}{2}$. The ratio of two quantities is usually expressed by, (:) being placed between them; thus, 2 : 8, which equals $\frac{8}{2}$ or 4.

A ratio cannot be a *concrete or denominate* number; neither is there a ratio between quantities of different denominations.

1. What is the ratio of 5 yards to 25 yards?
2. What is the ratio of 4 inches to 36 inches?
3. What is the ratio of 8 apples to 72 apples?
4. What is the ratio of 24 sheep to 96 sheep?
5. What is the ratio of 9 pounds to 108 lbs.?
6. What is the ratio of 4 feet to $16?
7. What is the ratio of 4 sheep to 24 horses?

PROPORTION.

ART. **172.** When two quantities have the same ratio as two other quanities, the *four* quantities are said to be in *Proportion.* Thus, the ratio of 8 bushels to 32 bushels, is the same as the ratio of $3 to $12.

PROPORTION is an equality of ratios of numbers compared together, *two* and *two.*

Quantities are shown to be in *proportion* by means of dots; for example, the above proportion is written,

$$\overset{\text{bush.}}{8} : \overset{\text{bush.}}{32} :: \overset{\$}{3} : \overset{\$}{12}$$

And is read 8 busels is to 32 bushels, as $3 is to $12.

REMARK.—The two dots placed between the *first* and *second.* also between the *third* and *fourth* terms, in the above proportion, are contractions of the sign of division (÷), the horizontal line being omitted. The four dots between the *second* and *third* are contracted from, and equivalent to, the sign of equality. Hence, the above was formerly written,

8 bush. ÷ 32 bush. = $2 ÷ $12. This expression indicates that the ratio is found by dividing the *first* term by the *second.* The English mathematicians have adopted this method of expressing the ratio of one number to another, while the French divide the *second* by the *first*, as previously directed.

The first two terms of proportion are called the *first couplet;* the second two terms, the *second couplet.*

The first term of each couplet is called the *Antecedent*, and the second term is called the *Consequent*.

The *first* and *fourth* terms of a proportion are called the *Extremes*, and the *second* and *third* terms are called the *Means*.

Since, in a proportion, the quotient obtained by dividing the *second* term by the *first*, is equal to the quotient obtained, by dividing the *fourth* term by the *third*, we can readily deduce the following

PROPOSITIONS.

1. *The product of the means is equal to the product of the extremes.* Therefore,

2. *If the product of the means be divided by one extreme, the quotient will be the other extreme.* Or,

3. *If the product of the extremes be divided by one mean, the quotient will be the other mean.*

4. *The fourth term of a proportion is equal to the third term, multiplied by the ratio of the first term to the second.*

SUGGESTION.—These propositions being understood, the pupil can readily determine the remaining term of a proportion, if any *three* of them be given.

SIMPLE PROPORTION.

ART. **173.** SIMPLE PROPORTION teaches the method of finding the *fourth term* of a proportion, by knowing the other three.

ART. **174.** In stating a question in Simple Proportion, the FIRST and SECOND *terms* must be of the same kind or denomination, and the THIRD term like the answer sought. If the answer is to be greater than the *third* term, the larger of the two remaining terms, must occupy the *second* place,—if smaller, the *first* place. Then proceed according to Proposition 2nd, or 4th.

1. If 12 bushels of wheat cost $21·60, what will 28 bushels cost?

EXPLANATION.—The answer sought is to be in dollars, therefore, we have the $21·60, for the *third* term. The answer is to be *greater* then the third term, because 29 bushels will cost more then 12 bushels; hence, we have the larger number, 29 for the second term and 12 for the first. Thus:

```
bush.   bush.     $
 12  :   29 :: 21·60
                 29
              -----
              19440
              4320
            -------
          12)626·40, the product of the means.
            -------
            $52·20, the other extreme, or 4th term.
```

The above question can as well be solved, by finding the ratio of the first to the second term. Thus: (See Prop. 4th).

OPERATION.

$$\frac{29}{\cancel{12}}\times\frac{\overset{1·80}{\cancel{21·60}}}{1}=\$52·20. \text{ Ans.}$$

2. What will 247 yards of cloth cost, if 25 yards cost $144·60?

3. What will 347 bushels of corn cost, if 84 bushels cost $66·40?

4. What will 384 bushels of wheat cost, if 35 bushels cost $30·80?

5. What will 347 boxes of raisins cost, if 312 boxes cost $436·12½?

6. If a man travel 485 miles in 18 days, how far at this rate will he travel in 125 days?

7. A garrison of 125 men has provisions for 35 days. How many of the men must be discharged, that the remainder may be supported for 125 days?

8. If 43 men can do a certain piece of work in 47½ days, how many days will it take 15 men to do the same?

9. A man bought cows, at $123·75 for 8. How much at the same rate would 35 cows cost.

10. Bought 23 pieces of delaine, each containing 47¾ yards, at the rate of $24·45 for 45 yards. How much did it all cost?

11. If a company of 190 men consume 54 barrels of flour in 6 weeks, how many barrels would it take to last them 1 year?

12. If \$273 in 3 years gives \$18·62½ interest, how long will it require to give \$184 interest?

13. If \$83 in two years 8 months give \$12·37½ interest, what sum in the same time will give \$375·12½ interest?

14. If 50 men build a wall 750 rods long in 8 days, how many men will be required to build 864·5 rods in half of the time?

15. If a railroad car go 23 miles in 45 minutes, how far will it go in 5 days of 10 hours each?

16. If in 247½ feet there are 15 rods, how many rods in 1 mile?

17. If 47 acres of land sell for \$684·48, what will be the cost of a farm containing 287·5 acres?

18. What will be the cost of 847·56 pounds of wool, if 84·5 pounds cost \$47·87½?

19. If 19 sheep yield 56½ pounds of wool, how many pounds will 387 sheep yield?

20. How many pounds of coffee can be bought for \$147·84, when 18 pounds cost \$1·93¾?

21. If a tree 25 feet 4 inches in height give a shadow of 50 feet 8 inches, what is the length of the shadow of a tree whose height is 84 feet 9 inches?

REMARK.—After stating the question, reduce the *first* and *second* terms to the same *denominate value*; also reduce the *third* term to its lowest denomination mentioned;—the answer will be of the same denomination.

OPERATION.

ft.	in.		ft.	in.		ft.	in.	
25	4	:	84	9	: :	50	8	: length of shadow required.

in. 304 : in. 1017 : : in. 608 length of shadow required.

```
      608
     ----
     8136
    6102
    -----
304)618336(2034 in. = 169 ft. 6 in. Ans.
    608
    ---
     &c.
```

22. If 8 horses eat 19 bushels 3 pks. of oats in a week, how much would 85 horses eat in the same time?

23. If 12 men in 6 weeks earn £145 10*s.* 9*d.*, how much can 84 men earn in half of the time?

24. If 14 bushels 2 pks. 4 qts. of clover seed are worth $67·12½, how much will 184 bush. 3 pks. 6 qts. cost?

25. If 15 horses in 4 days, consume 87 bush. 6 qts. of oats, how many horses will 610 bush. 1 pk. 2 qts. keep the same time?

26. If the transportation of 21 cwt. 147 miles, cost $23·87½, what will the transportation of 47 cwt. 3 qrs. 20 lbs. cost, 4 times as far?

27. If a person accomplish a certain piece of work in 242 days, by working 8 hrs. a day, in how many days will he accomplish the same work, by working 12¾ hours a day?

28. Allowing a person to perform a certain journey in 26 days, when the days are 10½ hours long; in what time ought he to accomplish the same journey, when the days are 13 hours long?

29. Allowing 13 A. 25 P. of land to produce 384 bush. 3 pks. of wheat, what number of bushels would be raised from a field containing 47 A. 3 R. 30 P., at the same rate?

30. An army of 4800 men had provisions for 8 months, one-sixth of the men having been killed in battle, how long ought the same provisions last the remainder?

31. If 18 head of cattle require 25 A. 3 R. of pasture ground, during the summer, how many acres ought 36 head to have for the same length of time?

32. Allowing the transportation of 25 T. 18 cwt. 20 lbs., a given distance, to cost $37·85; how much should be charged for the transportation of 18 T. 16 cwt. 3 qrs. 10 lbs. the same distance?

33. If a ship sail 247 leagues 1 mile 6 fur. in 15 days; in how many days would she sail 3000 miles?

34. A borrowed $250, which he kept 3 years 6 months. A subsequently, lends B $187½. How long ought B to keep this latter sum, in return for the accommodation he afforded A?

35. A merchant bought 3 pieces of cloth, each contain-

ing 23 yds. 3 qrs. for $495·75; and sold 54 yds. 2 qrs. of it for what it cost. How much did he receive for it?

36. If 8 yards 3 qrs. of cloth cost $34·50, how much will 83 E. English 3 qrs. cost?

37. If 5 E. French 4 qrs. of cloth cost $14·60, how much will 12 yds. 3 qrs. 2 nas. cost?

38. Allowing 14 horses to consume 65 bush. 3 pks. 5 qts. of oats in a week, how much would 74 horses consume in the same time?

39. If a person perform a certain journey in 14 days, by traveling $9\frac{5}{7}$ hours a day, how long will it take him to perform the same journey by traveling $12\frac{1}{2}$ hours a day?

40. What will be the cost of 9 cwt. 3 qrs. 20 lbs. of beef, if 8 cwt. cost $68?

41. If 36 sacks, each measuring 5 bushels, contain a given quantity of grain; how many sacks, each containing $3\frac{1}{3}$ bushels, will contain the same quantity?

42. Allowing 32 head of cattle to require 23 A. 3 R 25 P. of pasture ground, during the summer, how many acres will 145 cattle require for the same length of time?

43. If 4 men mow $7·97\frac{1}{2}$ A. of grass in a day, how many men will be required to mow 63·8 A. in half the time?

44. The capacity of a cistern is 3600 gallons, and is filled with water by a pipe which pours into it 10 gals. 3 qts. a minute. By a leakage, 1 gal. 2 qts. 1 pt. leaks out every minute during the time of filling. In what time will the cistern be filled?

45. If $\frac{5}{8}$ of an acre of land is worth $136, how much is $\frac{15}{16}$ of an acre worth?

REMARK.—Questions containing fractions, can be most conveniently solved by finding the ratio of the first to the second term, and then multiply the third term by it. (Art. 172, Proposition 4.)

OPERATION BY CANCELLATION.

$$\tfrac{5}{8} : \tfrac{15}{16} :: \$136 : \text{value sought.}$$

$$\frac{\cancel{8}}{\cancel{5}}\times\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{16}}}\times\frac{\overset{68}{\cancel{136}}}{1}=\$204 \text{ Ans.}$$

46. If $\frac{4}{5}$ of a farm is worth \$860, how much is $\frac{8}{9}$ of it worth?

47. If $\frac{14}{17}$ of a city lot is worth \$4800, how much is $\frac{3}{8}$ of it worth?

48. If $\frac{7}{8}$ of a barrel of flour is worth \$5·40, how much is $\frac{7}{12}$ of it worth?

49. What cost $16\frac{3}{4}$ pounds of tea, if $6\frac{3}{4}$ pounds cost \$8·55?

50. What length of board that is $16\frac{8}{13}$ inches in width, will be required to make a square foot?

51. Bought $15\frac{1}{5}$ yards of cloth for \$54·90; what will 25 yards 3 qrs. cost at the same rate?

52. If $\frac{5}{8}$ of a ship is worth \$34865, how much is the whole cargo worth?

53. If $\frac{3}{10}$ enough water run into a ship by a leak, in 1 day 9 hrs. 15 min., to sink her; how long before she will sink?

54. Bought $25\frac{2}{3}$ barrels of flour, at $\$6\frac{6}{11}$ a barrel, and paid for it with sheep, at $\$1\frac{1}{2}$ a head; how many sheep did it take?

55. If $6\frac{2}{3}$ barrels of sugar cost \$112.75, how much will $\frac{4}{5}$ of a barrel cost?

56. If $13\frac{3}{4}$ yards of cassimere cost $\$19\frac{1}{6}$, what will $5\frac{1}{2}$ yards cost?

57. If $2\frac{3}{4}$ barrels of beef cost \$20·75, how much will $1\frac{1}{2}$ barrels cost?

58. If 5 pounds of butter cost $62\frac{1}{2}$ cents, how much will $1\frac{3}{5}$ pounds cost.

59. If $\frac{2}{3}$of an apple cost $\frac{3}{4}$ of a cent, what will $\frac{7}{8}$ of an apple cost?

60. If it require 6 days for 10 men to build 360 rods of wall, how many men can in $\frac{1}{2}$ of the time build 720 rods of similar wall?

61. If 24 men in 8 days perform a certain piece of work, how many men will be necessary to accomplish 3 times as much work in $\frac{3}{4}$ of a day?

62. If it require 2 bushels of oats to feed 4 horses $\frac{1}{3}$ of a day, how many horses would it take to consume 144 bushels in $\frac{3}{5}$ of a day?

63. If a staff $9\frac{3}{4}$ feet long cast a shadow $12\frac{2}{3}$ feet, what is the height of that steeple the shadow of which, at the same time measures 285 feet?

64. If a steamship can sail 3000 miles in $9\frac{1}{2}$ days, how long, at the same rate of sailing, would she require to sail 24900 miles, the distance around the earth?

65. The diurnal rotation of the earth moves its equatorial portions about 24900 miles a day. (24 hours.) How far is that in each minute?

66. Admitting the earth to move in its orbit about the sun 597000000 miles, in 365 days 6 hours; how far on an average does it move in 1 minute?

67. If it require 35 yards of carpeting, which is $\frac{3}{4}$ of a yard wide to cover a floor, how many yards, which is $1\frac{1}{2}$ yards wide, will be necessary to cover the same floor?

COMPOUND PROPORTION.

ART. **175.** *Compound Proportion* teaches to find a required quantity in a proportion when it depends on more than three terms.

1. If 6 men can earn $72 in 10 days, by working 12 hours a day, how many dollars can 15 men earn in 8 days, by working 8 hours a day?

REMARK.—We will first solve this question by *analysis.*

ANALYSIS.—If 6 men in a certain time earn $72, 1 man in the same time will earn $\frac{1}{6}$ of $72 = $12; and 15 men will earn 15 times $12 = $180. If in 10 days 15 men earn $180, in 1 day they will earn $\frac{1}{10}$ of $180 = $18; and in 8 days they will earn 8 times $18 = $144. If in 8 days by working 12 hours a day, 15 men earn $144, by working 1 hour a day, they will earn $\frac{1}{12}$ of $144 = $12; and by working 8 hours a day they will earn 8 times $12 = $96.

SOLUTION BY CANCELLATION.

REMARK.—As the question is read the pupil will find it of assistance to write it down in the following manner, as he can then more easily remember the question and form the ratios. Taking the above example we proceed thus;—

men.	\$.	days.	hours.
6	72	10	12
15		8	8

$$\$\frac{\overset{12}{\cancel{72}}}{1} \times \frac{\overset{3}{\cancel{15}}}{\cancel{6}} \times \frac{\overset{4}{\cancel{8}}}{\underset{2}{\cancel{10}}} \times \frac{8}{\cancel{12}} = \$96. \text{ Ans.}$$

EXPLANATION.—If 6 men in a certain time earn \$72, 1 man will earn $\frac{1}{6}$ of \$72, and 15 men will earn $\frac{15}{6}$ of \$72. If 15 men earn $\frac{15}{6}$ of \$72 in 10 days, in 1 day they will earn $\frac{1}{10}$ as much, and in 8 days $\frac{8}{10}$ as much, which is $\frac{8}{10}$ of $\frac{15}{6}$ of \$72. If 15 men in 8 days earn $\frac{8}{10}$ of $\frac{15}{6}$ of \$72 by working 12 hours a day, by working 1 hour a day they will earn $\frac{1}{12}$ as much, and by working 8 hours a day $\frac{8}{12}$ as much, which is $\frac{8}{12}$ of $\frac{8}{10}$ of $\frac{15}{6}$ of \$72 = \$96.

2. If 12 men can mow 48 acres of grass in 8 days, by working 5 hours a day; how many acres can 56 men mow in 5 days, by working 12 hours a day?

3. If the wages of 36 men for 3 days be \$216; how many men in 4 days can earn \$192?

4. If 15 men can cut 280 cords of wood in 16 days, by working 9 hours a day, how many men will be required to cut 28 cords in 4 days, by working 6 hours a day?

5. If a man travel 240 miles in 14 days, by traveling 6 hours a day; how far can he travel in 18 days, by traveling $9\frac{1}{3}$ hours a day?

6. If 15 men in 9 days, by working 6 hours a day, build 36 rods of stone-fence; how many men will be required to build $133.\dot{3}$ rods in 14 days, by working 8 hours a day?

7. If 72 men in 18 days of 12 hours each, build a wall 162 rods in length, 12 feet high, and 9 feet thick; how many rods of wall that is 9 feet high, and 3 feet thick, can 40 men build in 8 days of 9 hours each?

8. If a marble slab 20 feet long, 5 feet wide, and 4 inches thick, weigh 850 pounds; what is the length of another slab that is 4 feet wide and 2 inches thick, that weighs 272 pounds?

9. If a family of 12 persons in 20 weeks and 4 days consume \$450 worth of provisions; how many persons will $\$3037\frac{1}{2}$ worth of provisions keep 45 weeks and 6 days?

10. If it require 264 yds. of cloth that is $1\frac{1}{4}$ yds. wide, to clothe 121 men; how many yards which is $1\frac{1}{2}$ yards wide will be required to clothe 220?

11. If 210 yds. of cloth, 1 yard wide, cost \$300, what will 140 yds. of similar cloth cost, that is 3 quarters wide?

12. If \$250 will in 7 months gain \$25, when the rate of interest is 10 per cent.; at what rate per cent., will \$750 in 9 months gain \$$67\frac{1}{2}$?

13. If a family of 24 persons consume \$120 worth of bread in $8\frac{2}{5}$ months, when flour is worth \$5 a barrel; how many dollar's worth will a family of 8 persons consume in 6 months, when flour is worth \$7 a barrel?

14. If 240 men, by working 8 hours a day, can in 81 days dig 256 cellars, each 24 feet long, 27 feet wide, and 18 feet deep; how many men can, in 27 days of 6 hours each, dig 18 cellars, each 40 feet long, 36 feet wide, and 12 feet deep?

15. If 24 men, by working 8 hours a day, can in 18 days dig a ditch 95 rods long, 12 feet wide, and 9 feet deep, how many men, by working 12 hours a day, for 24 days, will be required to dig a ditch 380 rods long, 9 feet wide and 6 feet deep, in a soil that is $1\frac{2}{3}$ times as difficult of excavation?

CONJOINED PROPORTION.

ART. **176.** *Conjoined Proportion* is a proportion in which each *antecedent* is equal in value to its *consequent*,—each *consequent* being of the same denomination as the preceding *antecedent*,—and the *first* and *last* terms, of the same denomination.

1. If 8 bushels of wheat are worth 3 cords of wood, and 9 cords of wood are worth 3 tons of hay, how many bushels of wheat are worth 6 tons of hay?

ANALYSIS.—If 3 tons of hay are worth 9 cords of wood, 1 ton is worth $\frac{9}{3}$ cords of wood. If 3 cords of wood are worth 8 bush.

of wheat, 1 cord is worth $\frac{8}{3}$ bushels. If 1 cord is worth $\frac{8}{3}$ bush. of wheat, $\frac{9}{3}$ cords (the value of 1 ton of hay), is worth $\frac{9}{3}$ times $\frac{8}{3}$ bushels of wheat, and 6 tons are worth 6 times $\frac{9}{3} \times \frac{8}{3}$ bushels $= 48$ bushels of wheat.

The conditions of the above question are expressed thus:

8 bushels = 3 cords of wood.
9 cords = 3 tons.
6 tons = how many bushels of wheat?

And may be solved by writing all the terms on the left of the equality, for the numerator of a compound fraction, and those on the right for the denominators. Thus:

$$\frac{6}{1} \times \frac{\overset{3}{\cancel{9}}}{\cancel{3}} \times \frac{8}{\cancel{3}} = 48 \text{ bushels of wheat.}$$

2. If 4 barrels of corn are worth 8 bushels of wheat, and 3 bushels of wheat are worth 5 bushels of rye, and 12 bushels of rye are worth 20 bushels of oats, how many bushels of oats are worth 12 barrels of corn?

3. A can do as much work in 3 days as B can in 6 days; and B as much in 5 days as C in 15 days. In how many days could A do as much work as C in 48 days?

4. If 48 yards of cloth in New York are worth 36 barrels of flour in Philadelphia; and 18 barrels of flour in Philadelphia are worth 24 bales of cotton in New Orleans; how many bales of cotton in New Orleans are worth 240 yards of cloth in New York?

5. If $12\frac{1}{2}$ yards of satin cost \$18·75; and \$10·25 will purchase 3 yards of broadcloth; and $6\frac{1}{4}$ yards of broadcloth are worth $18\frac{1}{2}$ yards of silk; how many yards of satin are worth 120 yards of silk?

COPARTNERSHIP.*

ART. **177.** COPARTNERSHIP is the association of two or more individuals in the transaction of business, who agree

* *Copartnership*, or *Fellowship*, is sometimes called PARTITIVE PROPORTION.

to share the profits and losses in proportion to the amount of capital they have in the partnership. Each individual thus associated is called a *Partner*. The partners together are called the *Company*, or *Firm*.

The *Capital Stock* is the amount of money employed in trade. The Dividend is the profit or loss to be shared.

1. A, B, and C entered into partnership. A put in \$240; B put in \$400; and C put in \$320. They gain \$192. How much is each man's gain?

OPERATION.

A's stock,		\$240
B's	"	400
C's	"	320
Capital stock,		960

Therefore, A owns $\frac{240}{960} = \frac{1}{4}$ of the entire stock.
B " $\frac{400}{960} = \frac{5}{12}$ " "
C " $\frac{320}{960} = \frac{1}{3}$ " "

Hence, A's gain is $\frac{1}{4}$ of \$192 = \$48
B's " $\frac{5}{12}$ of \$192 = \$80
C's " $\frac{1}{3}$ of \$192 = \$64

2. A, B, and C enter into partnership. A puts in \$360; B puts in \$440; and C puts in \$500. They gain \$780. How much is each man's gain?

3. A, B, C, and D, hired a pasture for \$12: A put in 12 sheep; B put in 16; C 18; and D 14. How much ought each to pay?

4. Four men traded in company and gained \$1680. A's stock was \$2000; B's \$1600; C's \$2400; and D's \$2000. How much is each man's gain?

5. A farm was purchased for \$7000, by A, B, and C. A furnished \$2500; B \$3000; and C \$1500. They receive \$560 rent yearly. How much of this rent should each receive?

6. A merchant employed 4 clerks, at the annual salaries of \$250, \$300, \$400, \$500, respectively. At the end of the year the merchant proving bankrupt, has but \$870 to

be divided proportionally among them. What will be the portion of each?

7. Divide $960 among three persons in such a manner that their shares shall be to each other as 5, 4, and 3 respectively?

8. Two persons form a partnership in trade, with a capital of $1500, of which the first contributed $940; and the second the remainder. They gain $640. How much is each one's share?

9. Divide the number 230 into three parts which shall be to one another as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

ANALYSIS.—the proportional terms being reduced to equivalent fractions having a common dènominator, we have $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$; and these fractions are to one another as their numerators 6, 8, and 9, since they have the same denominator. Hence we dĭvide the 230 into $6 + 8 + 9 = 23$ equal parts. Hence $\frac{6}{23}$, $\frac{8}{23}$, and $\frac{9}{23}$ of 230 respectively, gives the required numbers.

10. A, B, and C, found a purse containing $240, and agreed to share it in the proportion of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. How much should each receive?

11. A, B, and C enter into partnership: A puts in $160; B $280; and C $460. They lose $480. How much is each partner's loss?

12. A captain, mate, and 14 sailors, took a prize of $24600; of which the captain takes 11 shares; the mate 5 shares; and the remainder is equally divided among the sailors. How much did each receive?

13. Four partners, A, B, C, and D shipped 1280 sheep for Scotland; of which A owned 240; B 160; C 400; and D the remainder. In a severe storm they threw 320 of them overboard. How many sheep did D own, and how much was each partner's loss?

14. A, B, C, D, and E are to share $3045; A is to have a certain sum; B as much again as A; C as much as A and B together; D as much again as B; and E as much as D and A together. How much is each to have?

15. A, B, and C agree to contribute $620·62 towards building a church, which is to be situated 2 miles from A;

3 miles from B; and 5 miles from C. They also agree that their contributions shall be proportional to the reciprocals of their distances from the church. How much ought each to contribute?

16. A, B, and C contribute $3535·70 towards building an Academy, which is to be situated $1\frac{1}{2}$ miles from A; $1\frac{3}{4}$ miles from B; and $2\frac{1}{3}$ miles from C. They also agree that their contributions shall be reciprocally proportional to their distances from the Academy. How much did each contribute?

17. A, B, and C found a purse containing $280·70. They agreed to divide it in such a manner that A should have $\frac{2}{3}$ as much as B; and B $\frac{4}{5}$ as much as C. How much should A, B, and C receive respectively?

REMARK.—The pupil will find the proportional terms as follows:

$$\begin{aligned}
&\text{A's part} = \tfrac{2}{3} \text{ of B's,}\\
&\text{and} \qquad\qquad \text{B's} = \tfrac{4}{5} \text{ of C's}\\
&\text{Hence,} \qquad \tfrac{5}{4} \text{ of B's} = \text{C.}\\
&\text{Therefore, A's} = \tfrac{8}{12} \text{ of B's}\\
&\qquad\qquad \text{B's} = \tfrac{12}{12}\\
&\qquad\qquad \text{C's} = \tfrac{15}{12} \text{ of B's.}
\end{aligned}$$

Consequently we divide the $280·70 in proportion to the numbers 8, 12, and 15.

18. A, B, and C, in partnership lose $650. A's portion of the capital employed was $\frac{3}{4}$ of B's, and B's was $\frac{2}{3}$ of C's. What amount of loss should each sustain?

19. Four persons in a joint speculation gain $460, which is to be divided among them so that the second shall have $\frac{2}{3}$ as much as the first, and the second $\frac{3}{4}$ as much as the third. How much should each receive?

20. A farmer divided 1152 acres of land among his four sons, in such a manner that $\frac{2}{3}$ of John's number of acres equals $\frac{3}{4}$ of James'; $\frac{2}{3}$ of Jame's equals $\frac{3}{4}$ of Jackson's; and $\frac{2}{3}$ of Jackson's equals $\frac{3}{4}$ Joseph's number of acres. How many acres did each receive?

21. Three men A, B, and C, agree to reap a certain field of wheat, for $39·68; A and B calculate that they can do $\frac{4}{5}$ of the labor; A and C, that they can do $\frac{2}{3}$; and B and C that they can do $\frac{3}{5}$ of it. How much can each receive according to these estimates?

COMPOUND COPARTNERSHIP.

ART. **178.** When the stock of the several partners is employed in the trade for different periods of time, it is called *Compound Copartnership*. It is evident in such cases, that the *gain* or *loss* must be apportioned with reference to the *stock* and the *time* it has been employed in the business.

1. Three partners A, B, and C put money into trade as follows: A put in \$50 for 4 months; B, \$150 for 2 months; and C, \$250 for 3 months. They gained \$250. How much is each man's share of the gain?

OPERATION.

\$		m.		\$	
50	×	4	=	200	for 1 month.
150	×	2	=	300	" "
250	×	3	=	750	" "
				1250	Capital Stock.

EXPLANATION.—The preceding work becomes evident, by considering that the interest of \$50 for 4 months, is the same as the interest of \$200 for one month; &c. Therefore,

A's part of the entire stock $= \frac{200}{1250} = \frac{4}{25}$ of the whole.
B's " " " $= \frac{300}{1250} = \frac{6}{25}$ " "
C's " " " $= \frac{750}{1250} = \frac{3}{5}$ " "

Hence, A's gain $= \frac{4}{25}$ of \$250 = \$40
B's " $= \frac{6}{25}$ of \$250 = \$60
C's " $= \frac{3}{5}$ of \$250 = \$150.

2. A, B and C hire a pasture for \$240; A put in 16 cows for 10 weeks; B, 20 cows for 7 weeks; and C, 25 cows for 6 weeks. How much ought each to pay?

3. A, B, C, and D have together performed a piece of work, for which they receive \$266·40. A worked 16 days of 10 hours each; B worked 20 days of 12 hours each; C worked 14 days of 10 hours each; and D worked 15 days of 12 hours each. How much should each man receive?

4. A, B, C, and D engaged in partnership for 3 years. A advanced \$2500, B \$3·500, C and D, each \$3800. Nine months afterwards, A added \$600 to his stock; B

$350; C withdrew $480; and D withdrew $460. At the end of the 3 years, the profits were found to be $1200 How much is each one's share?

5. To gather a certain field of grain, A furnished 9 laborers 6 days; B 12 laborers for 4 days; and C 14 laborers for 5 days. For the whole work they received $54·85. How much should A, B, and C receive respectively?

6. An army, consisting of 3 generals, 5 colonels, 12 captains, and 6840 soldiers, took a prize of $89908·75, which they agree to divide among themselves in proportion to their pay and the time they have been in the army. The generals and colonels have been in the army 9 months; the captains 5 months; and the soldiers, 8 months; the generals have $60 a month; the colonels, $40; the captains, $15; and the soldiers, $10. How much ought each to receive?

ALLIGATION MEDIAL.*

ART. **179.** ALLIGATION MEDIAL teaches the method of finding the *average* value of a mixture when the several simples of which it is composed, and their values are known.

ART. **180.** Given the several ingredients and their respective values to find the average value of the compound

1. A farmer mixes together 10 bushels of oats, worth 40 cents a bushel; 15 bushels of corn, worth 50 cents a bushel; and 25 bushels of rye, worth 70 cents a bushel. What is the value of a bushel of the mixture?

OPERATION.

cts.	bush.	cts.
40 ×	10 =	400
50 ×	15 =	750
70 ×	25 =	1750
	50)	2900
		58 cts.

* Alligation Medial is sometimes called *Medial Proportion.*

EXPLANATION.

cts. bush.	cts.	
$40 \times 10 =$	400	10 bush. at 40 cts. a bushel is worth $4·00.
$50 \times 15 =$	750	15 bush. at 50 cts. a bushel is worth $7·50.
$70 \times 25 =$	1750	25 bush. at 70 cts. a bushel is worth $17·50.
50)	2900	$29 is the entire cost of the mixture, which being divided by 50, the whole number of bushels, gives 58 cents, the average value of 1 bushel.
	58 cts.	

2. A wine merchant mixed together 40 gallons of wine, at 80 cents a gallon; 25 gallons of brandy, at 70 cents a gallon; and 15 gallons of wine, at $1·50 a gallon. What is the value of a gallon of the mixture?

3. A grocer mixed 80 gallons of rum, worth 30 cents a gallon; 40 gallons of whiskey, worth 40 cents a gallon; and 20 gallons of water, at the usual price. What is the value of a gallon of the mixture?

4. A grocer mixed 120 pounds of sugar, worth 5 cents a pound; 150 pounds, worth 6 cents a pound; and 130 pounds, worth 10 cents a pound. What was the average value of a pound of the mixture?

5. A grocer sold 50 barrels of flour, at $7·20 a barrel; 70 barrels, at $8·20 a barrel; and 80 barrels, at $5·70 a barrel. How much on an average did he receive for a barrel?

ALLIGATION ALTERNATE.

ART. **181.** ALLIGATION ALTERNATE teaches the method of finding how much of several ingredients, the values of which are known, must be taken to make a compound of a certain value.

CASE I.

ART. **182.** Given the values of several ingredients, to make a compound of a given value. First, *Place the several values of the ingredients in a column, and the average value on the left of this column. Join with a curved line, the value of each ingredient that is less than the average value, with one or more that is greater; then place the difference between the value of each ingredient and the average value, opposite the price of the ingredient with which it is joined, and this dif*

ference, or the sum of these differences, (if there is more than one,) will be the quantity required of that ingredient.

1. How much sugar worth 6, 8, and 10 cents a pound, must be mixed together, so that a pound of the mixture may be worth 7 cents?

OPERATION.

$$7\left\{\begin{array}{l}6\\8\\10\end{array}\right. \begin{array}{l}=1+3=4 \text{ of the sugar, at } 6 \text{ cts. a pound.}\\ = \qquad\quad\;\; 1 \text{ `` `` `` } 8 \text{ cts. ``}\\ = \qquad\quad\;\; 1 \text{ `` `` `` } 10 \text{ cts. ``}\end{array}$$

EXPLANATION.—By taking one pound of each kind of the sugar, we shall receive on the 10 cent quality, 4 cents more than the average price of the mixture, and on the 6 cent quality 1 cent less than the average price. The gain and the loss on the different qualities of sugar are to be equal; therefore the quantities taken must be universally proportional to the gain and the loss on the respective qualities.

REMARK.—Questions of this kind admit of an *indefinite* number of answers. It is obvious if we take any other quantities which are to each other, as 4, 1 and 1; as 8, 2 and 2; 12, 3 and 3, &c., that they will each satisfy the condition of the question equally well.

It is evident that there may be as many answers of different ratios, as there are methods of connecting the several values of the ingredients. For example:

2. How many pounds of tea, at 5, 6, 9, and 12 shillings a pound, must be mixed, so that the mixture shall be worth 8 shillings a pound?

OPERATION.

1.
$$8\left\{\begin{array}{l}5\\6\\9\\12\end{array}\right. \begin{array}{l}=4\\=1\\=2\\=3\end{array}\ \text{Answer.}$$

2.
$$8\left\{\begin{array}{l}5\\6\\9\\12\end{array}\right. \begin{array}{l}=\qquad 4\\=1+4=5\\=\qquad 2\\=3+2=5\end{array}\ \text{Answer.}$$

3.
$$8\left\{\begin{array}{l}5\\6\\9\\12\end{array}\right. \begin{array}{l}=4+1=5\\=\qquad 1\\=3+2=5\\=\qquad 3\end{array}\ \text{Answer.}$$

4.
$$8\left\{\begin{array}{l}5\\6\\9\\12\end{array}\right. \begin{array}{l}=1\\=4\\=3\\=2\end{array}\ \text{Answer.}$$

5.
$$8\left\{\begin{array}{l}5\\6\\9\\12\end{array}\right. \begin{array}{l}=4+1=5\\=\qquad 4\\=\qquad 3\\=3+2=5\end{array}\ \text{Answer}$$

6.
$$8\left\{\begin{array}{l}5\\6\\9\\12\end{array}\right. \begin{array}{l}=\qquad 1\\=4+1=5\\=2+3=5\\=\qquad 2\end{array}\ \text{Answer.}$$

7.
$$8\left\{\begin{array}{l}5\\6\\9\\12\end{array}\right. \begin{array}{l}=1+4=5\\=4+1=5\\=3+2=5\\=2+3=5\end{array}\ \text{Answer.}$$

3. How much wine, at $1·10 per gallon, 60 cents per gallon, and 40 cents per gallon, must be mixed together, so that the mixture may be worth 80 cents per gallon?

4. How much wine, at $1·60 a gallon, and water, at the usual rate, must be mixed together, so that the compound may be worth $1·15 a gallon.

5. How much of each sort of grain, at 46, 54, 75, and 85 cents a bushel, must be mixed together so that the compound may be worth 65 cents a pound?

CASE II.

ART. 183. When one of the ingredients is limited to a given quantity.

1. A merchant wishes to mix 60 pounds of tea, worth $1·20, with three other kinds, worth $1·10, 70 cents, and 60 cents, a pound, respectively, so that the mixture may be worth $0·80 a pound. How many pounds of the last three kinds must be used?

OPERATION.

$$80\left\{\begin{array}{r}120\\110\\70\\60\end{array}\right.\begin{array}{l}=20\\=10\\=30\\=40\end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\}\times 3=\left\{\begin{array}{rlllll}60 & \text{pounds,} & \text{worth} & \$1{\cdot}20 & \text{a pound.}\\ 30 & \text{``} & \text{``} & \$1{\cdot}10 & \text{``}\\ 90 & \text{``} & \text{``} & \$0{\cdot}70 & \text{``}\\ 120 & \text{``} & \text{``} & \$0{\cdot}60 & \text{``}\end{array}\right.$$

EXPLANATION.—By Case 1, we obtain 20, 10, 30, and 40 pounds, respectively, which meets the requirements of the question, were neither of the quantities limited; but there is to be 60 pounds of that which is worth $1·20 a pound. We therefore, multiply the 20 opposite the $1·20 by such a number as will cause the product to become 60, which I find to be 3, and to preserve the value of the mixture the same per pound, we multiply all the other proportional quantities by the same number.

2. How much oats, at $·40 a bushel; barley, at $·45; and corn, at $·75, must be mixed with 60 bushels of rye, at $·85 a bushel, so that a bushel of the mixture may be worth $·60?

3. How much sugar, at 5, 8, and 10 cents a pound must be mixed with 64 pounds, at 12 cents a pound, so that the mixture may be worth 9 cents a pound?

4. A merchant has 40 pounds of tea, worth $1·50 a pound, which he wishes to mix with four other kinds, worth 95, 75, 60, and 40 cents a pound respectively. How much must he take of each of these four kinds, so that the mixture shall be worth 80 cents a pound?

CASE III.

Art. 184. When the whole mixture is to consist of a certain quantity.

1. A merchant has sugar worth 5, 6, 9, and 12 cents a pound;—with a mixture of these he wishes to fill a hogshead that shall contain 220 pounds. How much of each kind must he take, so, that the compound may be worth 8 cents a pound?

OPERATION.

$$8\left\{\begin{array}{l}5 \quad =4\\ 6 \quad =1\\ 9 \quad =2\\ 12 \quad =3\end{array}\right\} \quad 22=\left\{\begin{array}{lllll}88 & \text{pounds at} & 5 & \text{cents a} & \text{pound.}\\ 22 & `` & 6 & `` & ``\\ 44 & `` & 9 & `` & ``\\ 66 & `` & 12 & `` & ``\end{array}\right\}\text{Ans.}$$

$$10)\underline{220}$$

$$22 \quad \Big\{ \text{Ratio of the sum of the proportionate quantities to the number given.}$$

Explanation.—The sum of the proportionate quantities, (found by Case 1,) is 10; the whole number of pounds that is to compose the mixture, is 220; therefore, I must take $\frac{220}{10}$ times as much as the sum of these proportionals, which is 22 times each proportionate quantity.

2. How many gallons of water, brandy, and rum, must be taken, so as to make a mixture of 90 gallons, worth 80 cents a gallon; providing the water is of no value, the brandy being worth $1·20 a gallon, and the rum, 60 cents a gallon?

Remark.—Archimedes employed the above in detecting the fraud respecting the crown of Hiero, king of Syracuse. The king had ordered a crown of pure gold to be made; but suspecting his artist to have mixed alloy with it, he requested Archimedes to determine the fact without injuring the crown. To do this, Archimedes took a piece of pure gold, and another of alloy, each equal in weight to the crown, placing them respectively in a vessel filled with water and observing the quantity of water expelled by each he readily determined that the crown was composed of gold and alloy; also the exact proportion in which these ingredients were used.

3. Suppose the weight of the crown and of each mass to be 10 pounds; and that being placed in water, the alloy expelled ·92 lbs., the gold .52 lbs., and the crown ·64 lbs. Of how much gold, and of how much alloy, did the crown consist? Ans. 3 lbs. of alloy, and 7 lbs. of gold.

OPERATION.

$$64 \left\{ \begin{array}{l} \cdot 92 \\ \cdot 52 \end{array} \right\} \begin{array}{l} = \cdot 12 \\ = \cdot 28 \end{array} \times 25 = \left\{ \begin{array}{l} 3 \text{ pounds of alloy.} \\ 7 \quad \text{" \quad " gold.} \end{array} \right\} \text{Ans.}$$

$$\cdot 40)10 \cdot 00$$

$$25$$

CHAPTER VIII.

PERCENTAGE.

ART. **185.** The term PER CENT. is derived from the Latin words *per* and *centum*, which signify, *by the hundred.* Percent, therefore, is any sum or number on a hundred, whatever be the denomination. Thus, 5 per cent., signifies 5 for every hundred, or 5 *hundredths;* 8 per cent. signifies 8 for every hundred, or 8 *hundredths*, &c.

We have already learned that *hundredths* can be expressed either as a *common* or as a *decimal fraction;* thus, 5 hundredths $= \frac{5}{100} = .05$; 8 hundredths $= \frac{8}{100} = .08$, &c. In all our calculations in percentage, the rate per cent. is written in the *decimal* form.

ART. **186.** Percentage is extensively used in mercantile transactions, and more or less in the transactions of all other kinds of business; such as Assessment of Taxes, Insurance, Duties, Profit and Loss, Interest, Discount, &c., &c.

ART. **187.** Finding the percentage on any sum or quantity.

1. What is 5 per cent. of 225 barrels of sugar?

OPERATION.

$$\begin{array}{r} 225 \\ .05 \\ \hline 11{\cdot}25 \end{array}$$

It may also be solved thus; 5 per cent. is $\frac{5}{100} = \frac{1}{20}$ of the given quantity. Therefore, $\frac{1}{20}$ of 225 barrels = 11·25 barrels, is 5 per cent. of 225 barrels.

2. What is 6 per cent. of $140?
3. What is 8 per cent. of $340?
4. What is 35 per cent. of $380?
5. What is 47 per cent. of $160·35?
6. What is $12\frac{1}{2}$ per cent. of 146 yards of cloth?
7. What is $14\frac{3}{5}$ per cent. of 864 gallons of molasses?
8. What is $16\frac{2}{3}$ per cent. of 8472 barrels of flour?
9. A man, having $9684, lost by an investment $12\frac{1}{3}$ per cent. of it; how much had he remaining?
10. Bought 24 head of cattle at $25 a head, and sold them, at 25 per cent. advance; how much did I gain?
11. A merchant having $8645, gave 14 per cent. of it for silks; 28 per cent. of it for flour; 43 per cent. of it for broadcloth; and the remainder for sugar. How many dollars did he spend for each?
12. A farmer raising 987 bushels of wheat, gives 9 per cent. of it for gathering it; 10 per cent. of the remainder for thrashing; and 10 per cent. of what now remains for flouring. How much has he remaining?
13. A merchant bought 563 barrels of cider for $2837; and sold 45 per cent. of it, at $6·85 a barrel; 35 per cent. of it, at $7·12½ a barrel; and the remainder for what it cost. How much did he gain by the operation?
14. A speculator invested $8640 in a speculation, and lost 25 per cent.; he then invested the remainder in a speculation and gained 15 per cent.; he now invested this amount in speculation and gained 24 per cent. How much did he make by the operation?

INSURANCE.

ART **188.** INSURANCE is an agreement by which a

company, or individuals, obligate themselves to make good any *loss* or *damage* of property by fire, shipwreck, or other casualties.

The written agreement of indemnity issued by the *Insurers*, sometimes called the *underwriters*, to the persons whose property is insured, is called the POLICY.

The insurance is effected in consideration of a sum of money, called a PREMIUM, which is estimated at a certain rate per cent. on the amount insured, and is paid beforehand, to the insurers.

1. If A gets his ship and cargo insured for $86950, from New York to Liverpool, at 2 per cent.; how much will be the amount of the premium?

2. An insurance of $18640 was effected on the ship Baltic, at $2\frac{1}{2}$ per cent. How much did the premium amount to?

3. A dwelling, valued at $1485, was insured, at $\frac{3}{8}$ of 1 per cent. How much was the premium?

4. A steamboat, valued at $55016, has an insurance effected on $\frac{4}{5}$ of its value, at $3\frac{2}{3}$ per cent. How much is the premium?

5. A gentleman has his dwelling insured for $8640, at 29 cents on $100. What is the premium?

6. A person at the age of 40, effects an insurance on his life for 3 years for the sum of $12800, at the rate of $1·95 on $160 per annum. How much is the annual premium?

7. An individual, going to California with the intention of returning at the expiration of 3 years, effects an insurance of $9988 on his life, at $\frac{1}{2}$ of $\frac{2}{3}$ of $1\frac{1}{5}$ per cent. per annum. How much is the annual premium?

STOCKS, BROKERAGE AND COMMISSION.

ART. **189.** STOCKS are government funds, and the capital of incorporated institutions, such as banks, railroad and manufacturing companies, &c. Stocks are divided into shares, usually varying from $50 to $500 each, the market value of which is at times variable.

The *par value* of a share is its *original* cost. When it sells for more than its original cost, it is said to be *above par*, or at an *advance;* when it sells for less, it is *below par*, or at a *discount.*

The *rise* or *fall* in stocks is computed at a certain *per cent.* on the par value of the shares.

ART. **190.** BROKERAGE is the percentage paid to brokers, or dealers in stocks, money, bills of credit, and for the transaction of business.

ART. **191.** COMMISSION is the percentage paid to agents and commission merchants, for the purchase, sale, or care of property, and for the transaction of other business.

The rate *per cent.* of Brokerage or Commission, varies in different places, and depends upon the nature of the business transacted.

1. What will \$9864 par value of bank stock cost, at 18 per cent. advance?

REMARK.—Find 18 per. cent. of \$9864 and add it to the \$9864; the sum will be the amount required.

2. How much must be given for 25 shares in the Hudsoon River Railroad, at $12\frac{1}{2}$ per cent. advance, the shares being \$340 each?

3. What is the value of 27 shares of canal stock, at $18\frac{3}{4}$ per cent. advance, the shares being \$150 each?

4. How much will be the cost of 18 shares of bank stock, at $17\frac{3}{5}$ per cent. below par, the shares being \$240 each?

5. Bought 87 shares of a certain stock, at $13\frac{1}{2}$ per cent. below par, and sold the same, at $17\frac{3}{4}$ per cent. above par; how much did I gain, the original shares being \$184 each?

6. A gentleman paid a broker $\frac{5}{6}$ of 1 per cent. to invest \$84860 in government funds. How much was the brokerage?

7. A lady, having \$84847, paid an agent $1\frac{4}{5}$ per cent. commission a year, to take care of it for her. To how much did the commission annually amount?

8. An agent sells 8484 barrels of flour, at \$5·$87\frac{1}{2}$ a

barrel, and charges $1\frac{4}{5}$ per cent. commission. How much money must he pay to his employer after retaining his commission?

9. A merchant, having 8646 barrels, gave an agent $2\frac{3}{4}$ per cent. commission for selling it. How much did the merchant receive, after deducting the commission, if it were sold, at \$15·87½ a barrel?

10. A bank, failing, has in circulation \$984840, and is able to pay only 87½ per cent. How much money has the bank on hand?

11. A broker in New York exchanged \$87846 on a certain bank in Ohio, for $\frac{4}{5}$ per cent. How much was the brokerage?

12. A merchant in Cincinnati sends to a commission merchant in New York \$4536·42 to lay out in goods, after reserving his commission, which was 5 per cent. How much was his commission?

SOLUTION.—It will be understood that the agent receives 5 per cent. or $\frac{5}{100} = \frac{1}{20}$ of the money laid out for goods only, and not of his commission; therefore, if to $\frac{1}{20}$, (his commission,) we add $\frac{20}{20}$, (the money expended for goods,) we have $\frac{21}{20}$ equal to the sum of the commission and amount paid for the goods, which is \$4536·42. Hence, $\frac{1}{20}$ of the money paid for the goods, (which equals the commission,) is $\frac{1}{21}$ of \$4536·42 = \$206·02; and $\frac{20}{20}$, the amount paid for goods, is 20 times \$206·02 = \$4120·40.

13. A farmer sends to a broker \$84672, to be invested in government funds; after deducting the brokerage which was, at 4 per cent. on the amount invested. How much was invested, and how much was the brokerage?

14. A commission merchant receives \$14760 to purchase silk, with what remained after deducting his commission of 2½ per cent. How many pieces of silk did he buy, providing it was \$32 a piece?

CUSTOM HOUSE BUSINESS.

ART. **192.** DUTIES are taxes levied by government on goods imported.

These duties constitute the *revenue* of the country, and

are collected by Custom House officers, at the ports of entry.

Duties are *specific* or *ad valorem*. A *specific duty* is a certain sum imposed, on a ton, cwt., hogshead, bushel, yard, &c., regardless of the value of the commodity.

An *ad valorem* duty is a certain percentage, on the cost of the articles in the country from which they are imported.

Gross weight is the entire weight of the commodity, together with the cask, box, or bag, &c., containing it.

Tare is an allowance made for the weight of the cask, box, or bag, &c., containing the merchandise.

Draft is an allowance for waste. *Leakage* is an allowance of 2 per cent. for the waste of liquors in transportation.

Net weight is what remains after all deductions.

The usual allowance for *draft* is as follows:—

	lbs.	lb.		lbs.	lbs.
On	112	1	From	336 to 1120	4
From	112 to 224	2	"	1120 to 2016	7
"	244 to 336	3	More than	2016	9

NOTE.—The *draft* although it is not mentioned in the question, must be deducted, before the other stated allowances are made

In *ad valorem* duties no deduction is made.

ART. **193.** To find the specific duty on goods.

From the given quantity deduct all allowance, and multiply the remainder by the duty on a unit of the given quantity. The product will be the required duty.

1. What is the duty on 12 barrels of sugar, each weighing 175 pounds gross, at $1\frac{3}{4}$ cents a pound; tare 20 per cent. ?

OPERATION.

Gross weight,	2100 lbs.
Draft subtracted,	9 lbs.
	2091 lbs.
20 per ct. of 2091 lbs. tare,	418·2
Net weight,	1672·8 lbs. $\times$ ·01$\frac{3}{4}$ = \$29·274, duty.

2. What is the duty on 4 hogsheads of sugar, each weighing 1280 lbs. gross, at $2\frac{3}{4}$ cents a pound; tare 14 per cent.?

3. What is the duty on 420 bags of coffee, each weighing 240 pounds, at 3 cents a pound; tare, 3 per cent.?

4. What is the duty on 210 bags of coffee, the gross weight of each bag being 190 lbs., invoiced* at 5 cents a pound; the tare being 5 per cent., and the duty 25 per cent.?

5. When there is a duty on tea, of 10 cents a pound, what must be paid on 45 chests, each weighing 120 lbs.; tare 10 per cent.?

6. At 35 per cent. ad valorem, what will be the duty on 436 yards of satin, at $1·75 a yard?

7. What is the duty on 85 bags of pepper, each weighing 140 lbs. gross, invoiced at $6\frac{1}{2}$ cents a pound, at $3\frac{1}{2}$ per cent.; tare 5 per cent.?

8. What is the ad valorem duty, at $37\frac{1}{2}$ per cent., on 40 pieces of silk, each containing 35 yards, invoiced at $2·25 a yard?

9. What is the duty, at 18 cents a gallon, on 15 casks of wine, each containing 75 gallons?

10. What is the ad valorem duty, at $62\frac{1}{2}$ per cent., on a case of silks, invoiced at $95800?

11. What is the duty on 10 barrels of Spanish tobacco, each weighing 145 lbs. gross; tare 8 per cent., at $6\frac{3}{4}$ cents a pound?

12. What is the duty, at 40 per. cent. ad valorem, on 15 cases of French broadcloth, each case containing 25 pieces, and each piece 35 yards, invoiced at $3·95 a yard?

Assessment of Taxes.

Art. **194.** Taxes are moneys paid by the people, to defray government expenses. Taxes are assessed on the citizens in proportion to their *real estate*† and *personal pro-*

* An invoice is a list of the articles imported, and the cost thereof.
† *Real Estate* is immovable property, as lands, houses, &c.

perty,* except the *poll-tax*, which is so much for each male individual over 21 years of age, regardless of his property.

Before taxes are assessed, an inventory of all taxable property in the state, county, or town in which they are to be paid, must be made; together with a list of the number of individuals liable to pay a poll-tax.

Then, from the sum to be raised, subtract the amount of the poll-taxes, and divide the remainder by the amount of taxable property, which will give the sum to be paid on $1, and multiply this sum, expressed in decimals, by each man's inventory, and the product will be the tax on his property.

1. A tax of $840·75 is to be raised in a town containing 65 polls. The taxable property in the town amounts to $48000. Each poll-tax is 0.75. What will be A's tax, whose property is valued at $375, and who pays one poll?

Ans. $6·94 nearly.

OPERATION.

$0·75	$840·75 the tax to be raised.
65	48·75 the amount of poll-taxes.
375	$792·00 Remainder.
450	
$48·75 the amount of poll-taxes.	

$\frac{792}{48000} = \cdot0165$ the tax on $1.

$375 \times \cdot0165 = \$6\cdot1875$ tax on property.

$\cdot75$ poll-tax.

$6·9375 Amount.

Explanation.—We find the amount of the poll-taxes to be $65 \times \$\cdot75 = \$48\cdot75$, which we deduct from $840·75, and have $792. If on $48000 there are $792 taxes to be paid, on $1 there must be paid $\frac{1}{48000}$ of $792 = $0·0165, and on $375, A's inventory, 375 times $·0165 = $6·1875. This being increased by 1 poll-tax = $6·94, A's tax.

Remark.—After having determined the amount to be paid on $1, the work of determining the tax of each particular individual may be facilitated by forming the following table. If we desire to find the tax on $500, remove the decimal point in the tax on $5 two places to the right, and we have $8·25, the

* *Personal Property* is that which is movable, as money, furniture, cattle, &c.

tax on $500. The pupil will readily understand the application of this table, and will also perceive that it is the best one that can be formed, although not the one usually given by arithmeticians.

	$		$		$		$
Tax on	1	is	·0165	Tax on	11	is	·1815
"	2	"	·033	"	12	"	·198
"	3	"	·0495	"	13	"	·2145
"	4	"	·066	"	14	"	·231
"	5	"	·0825	"	15	"	·2475
"	6	"	·099	"	16	"	·264
"	7	"	·1155	"	17	"	·2805
"	8	"	·132	"	18	"	·297
"	9	"	·1485	"	19	"	·3135
"	10	"	·165				

2. By the above table, what would be the tax on $984, there being 1 poll?

3. If I pay 4 polls, and am worth $1718·40, how much is my tax?

4. How much is that man's tax, who pays 2 polls, and is worth $284·85?

5. How much is that man's tax, who pays 3 polls, and is worth $8972·50?

6. How much is that man's tax, who pays 5 polls, and is worth $1784·84?

7. How much is that man's tax, who is worth $1984·35, and pays 2 polls?

Profit and Loss.

ART. **195.** *Profit and Loss* refer to the amount which the merchant or other business man, gains or loses in business transactions.

1. Bought 47 barrels of sugar, at $14·87½ a barrel, and sold it at $16·12½ a barrel. How much did I gain?

2. Bought 184 cords of wood, at $3·18¾ a cord, and sold it, at $4·37½ a cord. How much did I gain by the operation?

3. Bought 387 barrels of flour, at $5·93¾ a barrel, and sold it, at $6·75 a barrel. How much was the gain?

4 Bought 16 barrels of sugar, each containing 195

pounds, at \$13·84 a barrel, and sold it for \$.09$\frac{4}{5}$ a pound. How much was the gain?

5. Bought flour, at \$6·20 a barrel, and sold it so as to gain 20 per cent.; for how much did I sell it a barrel?

SOLUTION.—If on 100 cents I gain 20 cents., on 1 cent. I will gain $\frac{1}{100}$ of 20 cents $= \frac{20}{100} = \frac{1}{5}$ of a cent. Therefore, I gain $\frac{1}{5}$ of what it cost, $\frac{1}{5}$ of \$6·20 = \$1·24, which added to the cost equals \$7·44, what I must sell it for. Or,

Find 20 per cent. of \$6·20; thus, \$6·20 × ·20 = \$1·24; to which add the cost, and we have \$7·44, what it must be sold for a barrel.

6. Bought broadcloth, at \$5·85 a yard, and sold it so as to gain 25 per cent.; for how much did I sell it a yard?

7. A horse was bought for \$285·75; for how much must it be sold to gain 20 per cent.?

8. A merchant bought 185 barrels of pork, at \$18·95 a barrel; but it becoming damaged, he was obliged to lose 35 per cent. on the sale of it. How much did he receive for it all?

9. A merchant bought 25 pieces of silk, each containing $37\frac{1}{2}$ yards, for \$675·40, and sold it so as to gain $33\frac{1}{3}$ per cent. For how much did he sell it a yard?

10. A quantity of butter was bought for \$150, and sold for \$200; how much was the gain per cent.

SOLUTION.—On \$150 the gain is \$200—\$150 = \$50. If on \$150 there is a gain of \$50, on \$1, the gain will be $\frac{1}{150}$ of \$50 $= \frac{50}{150} = \frac{1}{3}$ of a dollar, or $33\frac{1}{3}$ per cent.?

11. A gentleman invested 4280 in speculation, and at the end of a year realized \$5350; how much per cent. did he gain?

12. A horse was bought for \$240, and sold for \$400; how much was the gain per cent?

13. A gentleman sold a horse for \$150, and thereby gained 25 per cent.; how much did the horse cost him?

SOLUTION.—If he gained 25 cents on 100 cents, on 1 cent, he gained $\frac{25}{100} = \frac{1}{4}$ of a cent. Therefore, he gained $\frac{1}{4}$ of what the horse cost him, which added to $\frac{4}{4}$, the cost of the horse, $= \frac{5}{4}$ of

the cost of the horse, which is equal to \$150, what he sold the horse for; and $\frac{1}{4}$, $= \frac{1}{5}$ of \$150 $=$ \$30; and $\frac{4}{4}$, the cost of the horse, $=$ 4 times \$30 $=$ \$120.

14. A quantity of salt was sold for \$864, which was $33\frac{1}{3}$ per cent. more than it cost him; how much did it cost him?

15. If in 1 year the principal and interest of a certain note, at $9\frac{2}{3}$ per cent., amount to \$12000. How much was the face of the note?

16. A quantity of rye was sold for \$1896, which was $18\frac{3}{4}$ per cent more than it cost. How much did it cost?

PRACTICAL QUESTIONS IN PROFIT AND LOSS.

1. If I buy 218 yards of broadcloth, at \$4·64 a yard, and sell it at \$6·95$\frac{1}{2}$ a yard; how much do I gain by the operation?

2. If I pay \$846 for a quantity of wheat; for what must I sell it to gain $23\frac{1}{2}$ per cent.?

3. Sold 149 barrels of cider, at \$4·87$\frac{1}{2}$ a barrel, and thereby gained $37\frac{1}{2}$ per cent. What did it cost a barrel?

4. Bought 480 gallons of molasses, at 28 cents a gallon, and sold it for \$168. How much did I gain per cent.

5. A house that cost \$1500, was sold for \$1250. What was the loss per cent.?

6. A farm that cost \$6500, was sold for \$9100. What was the gain per cent.?

7. Bought raisins, at \$3 a box; how much will be the loss per cent. if I sell it, at \$2·50 a box?

8. Sold 280 yards of cloth for \$700, and thereby gained 25 per cent., for how much should I have sold it a yard, to lose 20 per cent.?

9. If I sell 15 yards of broadcloth for \$66, and thereby gain 10 per cent., how ought I to have sold it a yard to have lost 25 per cent.?

10. A quantity of wheat was sold for \$360·90, which was 10 per cent. less than its original cost.; what would have been the gain per cent. if it had been sold for \$450·15?

11. Sold 45 boxes of damaged raisins for \$103·50, which was at a loss of 8 per cent.; how should I have sold them a box to have gained $\cdot\dot{3}$ per cent. ?

12. A house and lot was sold for \$2700, which was 8 per cent. more than its value; what would have been the gain per cent. if it had been sold for \$2833·$\dot{3}$?

13. A mechanic built a house for \$1980, which was 10 per cent. less than what it was worth; how much should he have received for it so as to have made $37\frac{1}{2}$ per cent. ?

14. A gentleman sold two farms for \$3680 a piece; for one he received 25 per cent. more than its value; and for the other, 25 per cent. less than its value. Did he gain or lose by the operation, and how much ?

15. A merchant sold two boxes of goods for \$540 a piece; on one he gained 20 per cent. and on the other he lost 20 per cent. Did he gain or lose by the operation, and how much ?

16. A speculator sold two building lots for \$1200 a piece, on one he received $37\frac{1}{2}$ per cent. more than it was worth, and on the other 25 per cent. less than what it was worth. Did he gain or lose, and how much ?

SIMPLE INTEREST.

ART. **196.** INTEREST is money due for the use of money or its equivalent; and is estimated at a certain rate *per cent. per annum*, which is generally fixed by law.

The PRINCIPAL is the sum on which the interest is paid.

The AMOUNT is the sum of the *principal* and interest.

By 7 per cent. is meant 7 cents on 100 cents, \$7 on \$100, or 7 *on* 100, *whatever* be the *denomination.*

The rate per cent. is different in different States. In the State of New York it is 7 *per cent.*, and in the New England States it is 6 *per cent.*, &c.

CASE I.

1. What is the interest on \$680 for 6 years, at 7 per cent. ?

OPERATION.

$$\begin{array}{l} \$\cdot 07 \text{ int. of } \$1 \text{ for 1 year.} \\ \quad\ \ 6 \\ \hline \$\cdot 42 \text{ `` `` `` `` 6 years.} \\ \quad 680 \\ \hline \quad 3360 \\ 252 \\ \hline \end{array}$$

$285·60 int. of $680 for 6 years, at 7 per cent.

EXPLANATION.—If the interest of $1 for 1 year is 7 cents, the interest for 6 years will be 6 times 7 cents, equal to 42 cents. If the interest of $1 is 42 cents, the interest of $680 is 680 times $·42, equal to $285·60.

REMARK.—Much care should be taken to keep the decimal point in its proper place.

2. What is the interest of $470 for 4 years, at 7 per cent.?

3. What is the interest of $683 for $2\frac{1}{2}$ years, at 6 per cent.?

4. What is the interest of $846·47 for $3\frac{2}{3}$ years, at 7 per cent.?

5. What is the interest of $86·42 for $3\frac{1}{3}$ years, at 8 per cent.?

6. What is the interest of $224·45 for $6\frac{2}{3}$ years, at 6 per cent.?

7. What is the interest of $249·98 for $4\frac{3}{4}$ years, at 7 per cent.?

8. What is the interest of $1·84 for 7 years, at $5\frac{1}{2}$ per cent.?

9. What is the interest of $16·37$\frac{1}{2}$ for $3\frac{4}{5}$ years, at $6\frac{1}{2}$ per cent.?

10. What is the interest of $215·12$\frac{1}{2}$ for $4\frac{5}{6}$ years, at $8\frac{2}{3}$ per cent.?

CASE II.

To find the interest on any sum of money, for any given time, at 6 per cent.

☞ The interest of \$1 for 12 months, (or 1 year,) is \$0·06, which is equal to *half the number* of months. Therefore, *half of the number of months equals the interest, in* CENTS, *of* \$1 *for the same number of months.* The interest of \$1 for 12 months being \$0·06, the interest for 2 months, ($=\frac{2}{12}$, or $\frac{1}{6}$ of a year,) is $\frac{1}{6}\times$ \$0·06 = \$0·01: Again, 6 days is $=\frac{6}{60}$, or $\frac{1}{10}$ of 2 months of 30 days each; therefore, the interest of \$1 for 6 days is $\frac{1}{10}\times$ \$0·01 = \$0·001. Therefore, *one-sixth of the number of days equals the interest, in* MILLS, *of* \$1 *for the same number of days.*

Hence, to find the interest of \$1 for any given time, at 6 per cent.:

Call half the number of months CENTS, *and one-sixth the number of days,* MILLS. The interest of \$1 being found, multiply it by the number of dollars in the given principal, and the product will be the interest required.

1. What is the interest of \$587·36 for 2 years 4 months and 24 days at 6 per cent.?

OPERATION.

2 years 4 months = 28 months. Calling the half of the 28 months cents, we have;

\$.14, int. of \$1 for 2 years and 4 months, at 6 per ceni.

Calling $\frac{1}{6}$ of the 24 days mills, we have;

\$·004, int. of \$1 for 24 days, at 6 per cent.

Hence, \$ ·14, int. of \$1 for 2 yrs. 4 mo. at 6 per cent.
·004, " " " " 24 days, at 6 per cent.

\$ .144, } gives the int. of \$1 for 2 yrs. 4 mo. 24 days, at
587·36 } 6 per cent.

864
432
1008
1152
·720

\$84·57984 } int. of \$587·36 for the given time and the given rate per cent.

2. What is the interest of \$84·25 for 1 year and 6 months, at 6 per cent.?

3. What is the interest of $184·50 for 3 years and 8 months, at 6 per cent.?

4. What is the interest of $273·84 for 2 years and 9 months, at 6 per cent.?

5. What is the interest of $847·80 for 4 years 7 months and 12 days, at 6 per cent.?

6. What is the interest of $684·45 for 3 years 8 months and 18 days, at 6 per cent.?

7. What is the interest of $849·95 for 5 years 5 months and 6 days, at 6 per cent.?

Remark.—To find the Amount add the *principal* and *interest* together.

8. What is the amount of $684·45 for 2 years 3 months and 18 days, at 6 per cent.?

9. What is the amount of $483·85 for 3 years 5 months and 24 days, at 6 per cent.?

10. What is the amount of $101·01 for 6 years 8 months and 14 days, at 6 per cent.?

11. What is the amount of $849·87½ for 2 years 9 months 25 days, at 6 per cent.?

12. What is the interest of $88·88 for 4 years 11 months and 22 days, at 6 per cent.?

CASE III.

Art. **197.** To find the interest on any given sum for any given time, at any given rate per cent. First, *find the interest of* $1 *for the given time, at* 6 *per cent.*, (See Case 2;) *then take as many* SIXTHS *of the interest as are equal to the given per cent.*, which will be the interest of $1 for the given time and rate per cent.; *then multiply this interest by the principal.*

If the interest is at 7, 9, or 11 per cent., &c., it is evident that, if to the interest of $1, at 6 per cent., we add its $\frac{1}{6}$, $\frac{3}{6}$, or $\frac{5}{6}$, &c., it will give the interest of $1, at 7, 9, or 11, per cent., &c., respectively. If the interest is at 2, 3, or 5 per cent., &c.; then, from the interest of $1, at 6 per cent., we must take its $\frac{4}{6}$, $\frac{3}{6}$, or $\frac{1}{6}$, &c., which will give the interest of $1, at 2, 3, or 5 per cent., &c., respectively.

1. What is the interest of $260 for 1 year 6 months and 18 days, at 8 per cent.?

OPERATION.

$·093, int. of $1 for the given time, at 6 per cent.
·031, *two-sixths* of the above interest.

$·124, int. of $1 for the given time, at the given rate per cent.
260

7440
248

$32·240 interest required.

2. What is the interest of $84·75 for 2 years and 10 months, at 7 per cent.?

3. What is the interest of $65·65 for 1 year 11 months and 23 days, at 7 per cent.?

4. What is the interest of $384·37½ for 2 years and 9 months and 16 days, at 8 per cent.?

5. What is the interest of $284·95 for 3 years 8 months and 20 days, at 7 per cent.?

6. What is the interest of $847·37½ for 4 years 7 months, at 7 per cent.?

7. What is the interest of $1284·62½ for 2 years 10 months and 4 days, at 7 per cent.?

8. What is the interest of $884·88 for 4 years 5 months and 5 days, at 5 per cent.?

9. What is the interest of $847·65 for 5 years 9 months and 15 days, at 5 per cent.?

10. What is the interest of $8484·84 for 7 years 4 months and 20 days, at 4 per cent.?

11. What is the interest of $1465·87½ for 8 years 8 months and 8 days, at 3 per cent.?

REMARK.—If the principal be given in English money, reduce the shillings, pence and farthings, to the decimal of a pound; then proceed as in Federal money.

12. What is the interest of £84 10*s*. 6*d*. for 3 years and 8 months, at 7 per cent.?

13. What is the interest of £145 15*s.* 8*d.* for 2 years and 6 months, at 7 per cent.?

14. What is the interest of £284 12*s.* 10*d.* for 1 year 8 months and 12 days, at 8 per cent.?

15. What is the interest of £384 10*s.* 6*d.* for 3 years 8 months and 24 days, at 7 per cent.?

ART. **198.** The following method of computing interest avoids the use of fractions, and may, therefore, be preferred by some.

We shall in accordance with general usage, reckon 30 days to the month, and 12 months to the year.

1. What is the interest of $460 for 2 years 7 months, at 9 per cent.?

OPERATION.

```
      $460
       ·09
      ----
    $41·40, interest for 1 year.
        31
    ------
      4140
     12420
    ------
12)1283·40
    ------
   $106·95 interest required.
```

EXPLANATION.—I find the interest of $460 for 1 year, (12 months.) at 9 per cent. to be $41·40. In the given time there are 31 months. If the interest of $460 for 12 months is $41·40, for 1 month it is $\frac{1}{12}$ as much: and for 31 months, it is 31 times $\frac{1}{12} = \frac{31}{12}$ of $41·40 = $106·95. Hence, to find the interest of any sum, when the time is given in years and months,

Multiply the interest of the principal for 1 *year by the number of months and divide the product by* 12.

For a similar reason, when the time is given in years, months and days,

Multiply the interest of the principal for 1 *year by the number of days and divide the product by* 360, *the quotient will be the interest required.*

☞ It may be inferred from what has already been remarked, that none of the preceding methods of computing interest is strictly correct; however they are in general use. The following correct method is adopted by many bankers and brokers.

Art. 199. *Multiply the interest of the principal for* 1 *year by the* EXACT *number of days it has been on interest, and divide the product by* 365, *the quotient will be the interest required.*

2. What is the interest of $720 for 2 years 9 months and 25 days, at 8½ per cent.?

OPERATION.

```
                              $720
                               ·08½
                              ----
                              5760
                               360
                              ----
                             61·20 interest for 1 year.
2 years 9 months 25 days =    1015 days.
                             -----
                             30600
                             6120
                           6120
                           -------
                     360)62118·00($172·55 interest required.
                         360
                         ----
                         2611
                         2520
                         ----
                           918
                           720
                           ---
                           &c.
```

REMARK.—The above question is solved by the method given under Art. 198. The pupil should also solve the same and the following questions by Art. 199, that he may discover the difference between the correct and the incorrect method of calculation.

3. What is the interest of $14·40 for 3 years 7 months, at 7 per cent.?

4. What is the interest of $25·20 for 4 years 5 months and 17 days, at 9 per cent.?

5. What is the interest of $100·80 for 5 years 9 months and 20 days, at 5 per cent.?

6. What is the interest of $201·60 for 3 years 7 months and 25 days, at 6 per cent.?

7. What is the interest of $403·20 for 3 years 8 months and 8 days, at $8\frac{1}{2}$ per cent.?

8. What is the interest of $806·40 for 4 years 5 months and 27 days, at $3\frac{1}{2}$ per cent.?

9. What is the interest of $720 for 6 years 6 months and 6 days, at $5\frac{1}{2}$ per cent.?

10. What is the interest of $1440 for 1 year 9 months and 15 days, at $8\frac{1}{2}$ per cent.?

ART. 200. Many prefer to calculate interest by *multiplying the principal by the rate per cent., and this product by the number of years; then add the interest for the months and days, found by means of aliquot parts, to the last product.*

TABLE.

ALIQUOT PARTS OF A YEAR OR MONTH.

mo.		yr.	days.		mo.
1	=	$\frac{1}{12}$	1	=	$\frac{1}{30}$
2	=	$\frac{1}{6}$	2	=	$\frac{1}{15}$
3	=	$\frac{1}{4}$	3	=	$\frac{1}{10}$
4	=	$\frac{1}{3}$	5	=	$\frac{1}{6}$
5	=	$\frac{5}{12}$	6	=	$\frac{1}{5}$
6	=	$\frac{1}{2}$	10	=	$\frac{1}{3}$
7	=	$\frac{7}{12}$	12	=	$\frac{2}{5}$
8	=	$\frac{2}{3}$	15	=	$\frac{1}{2}$
9	=	$\frac{3}{4}$	18	=	$\frac{3}{5}$
10	=	$\frac{5}{6}$	20	=	$\frac{2}{3}$
11	=	$\frac{11}{12}$	&c., &c.		

REMARK.—It is customary in the calculation of interest, to reckon 30 days to the month, and 12 months to the year, although this is not true, as some of the months contain more, and one of them less than 30 days; hence, the results obtained in these calculations are sometimes too large, and at other times too small, yet they are sufficiently correct for all practical purposes. But should it be desired to compute the interest with more accuracy, it may be done by finding the number of days the principal has been on interest, by the table, and consider this number of days as such a part of 365, a year. (*See* ART. 199.)

1. What is the interest of $240·50 for 3 years 4 months and 15 days, at $8\frac{1}{2}$ per cent.?

OPERATION.

	$ 240·50
	·08½
	19·2400
	1·2025
4 mo. = ⅓ yr.	$20·4425 interest for 1 year.
	3
15 days = ½ mo.	$61·3275 interest for 3 years.
or ⅛ of 4 mo.	6·8141⅔ interest for 4 months.
	·8517⅝ interest for 15 days.
	$68·9934+ interest required.

1. What is the interest of $1200·12½ for 6 years and 4 months, at 5 per cent.?

2. What is the amount of $87·95 for 2 years 3 months and 20 days, at 7 per cent.?

3. What is the amount of $47·84 for 4 years 1 month and 25 days, at 6⅗ per cent.?

4. What is the amount of $144·44 for 3 years 6 months and 18 days, at 7½ per cent.?

5. What is the amount of $650·30 for 3 years 7 months and 12 days, at 7⅔ per cent.?

6. What is the amount of $460·40 for 4 years 8 months and 15 days, at 8¾ per cent.?

7. What is the interest of $640·12½ from Jan. 24th, 1840, to March 28th, 1841, at 6½ per cent.?

8. What is the interest of $485·93¾ from Feb. 5th, 1842, to Aug. 20th, 1844, at 7½ per cent.?

9. What is the interest, at 5¾ per cent., of $846·84, from Jan. 8th, until Nov. 20th?

10. What is the interest, at 8⅔ per cent., of $384·25 from Jan. 12th, 1853, to April 4th, 1854?

11. What is the amount of $144·45 from Aug. 29th 1852, to Nov. 28th, 1853?

12. What is the interest of $1200·12½ from May 22nd 1852, to Sept. 9th, 1854?

13. What is the interest, at 9⅘ per cent. of $145·50 from July 14th, 1851, to Sept. 9th, 1853?

14. What is the interest of \$846·80 from Sept. 8th, 1847, to Aug. 8th, 1853?

15. What is the interest of \$784·93$\frac{3}{4}$ from Feb. 2nd, 1850, to April 24th, 1854?

PROBLEMS IN INTEREST.

ART. **201.** The PRINCIPAL, TIME, RATE PER CENT., and INTEREST, have such a relation to one another, that any three of them being given, the remaining one can readily be found by analysis.

NOTE.—For a complete analysis of *Interest, Discount* and *Percentage* of every description, see the last chapter in the "American Intellectual Arithmetic."

PROBLEM 1.—Given the rate per cent., time and interest to find the *principal.*

1. What principal will, in 2 years and 6 months, at 6 per cent. give \$6·18 interest?

SOLUTION.—2 years and 6 months equals $\frac{5}{2}$ years. The interest of \$1 for 1 year is 6 cents, and for $\frac{1}{2}$ of a year, $\frac{1}{2}$ of 6 cents $=$ 3 cents; and for $\frac{5}{2}$ years, 5 times 3 cents $=$ 15 cents. If the interest on 100 cents is 15 cents, on 1 cent it is $\frac{1}{100}$ of 15 cts. $= \frac{15}{100} = \frac{3}{20}$ of a cent. Therefore, $\frac{3}{20}$ of the principal equals the interest, which is \$6·18; and $\frac{1}{20}$ of the principal $=$ $\frac{1}{3}$ of \$6·18 $=$ \$2·06, and $\frac{20}{20}$, the principal $=$ 20 times \$2·06 $=$ \$41·20.

REMARK.—A similar method of analysis without further illustration can be readily applied by the pupil to all the following problems.

The interest on any sum is as many times greater than the interest on \$1, as that sum is greater than \$1. Hence, questions like the above may be solved by *dividing the given interest by the interest of* \$1, *at the given rate per cent. for the given time.*

2. What principal will, in 4 years and 9 months, at 8 per cent., give \$19·38 interest?

3. What principal will, in 3 years 8 months and 15 days, at 7 per cent., give \$177·55$\frac{1}{2}$ interest?

4. What principal will, in 4 years 9 months and 18 days, at 6 per cent., give \$86·688 interest?

5. What principal will, in 10 years 10 months and 20 days, at $6\frac{4}{5}$ per cent., give \$14·11653 interest?

PROBLEM 2.—Given the principal, the rate per cent., and the interest, to find the *time.*

1. In what time will \$26, at 6 per cent., give \$1·95 interest?

ART. **202.** The interest on a given principal is in proportion to the *time*, other things remaining the same. Hence, to find the *time*, the other three things being given; *Divide the given interest by the interest of the given principal, at the given rate per cent. for* 1 *year;* or solve it by Analysis.

2. In what time will \$300, at 8 per cent., give \$20 interest?
3. In what time will \$90·25, at 6 per cent., give \$4·75 interest?
4. In what time will \$284·75, at $5\frac{3}{4}$ per cent., give \$18·75 interest?
5. In what time will \$174·95, at $7\frac{4}{5}$ per cent., give \$34·87$\frac{1}{2}$ interest?

PROBLEM 3.—Given the principal, the time, and the interest, to find the *rate per cent* ?

1. The interest of \$65, for 10 months is \$3·25. What is the rate *per cent.* ?

ART. **203.** The interest on a given principal is in proportion to the *rate per cent.*, other things remaining the same. Hence, to find the rate per cent., the remaining three things being given; *Divide the given interest by the interest of the given principal, at* 1 *per cent., for the given time.*

2. The interest of \$120 for 2 years 9 months and 12 days, is \$13·36. What is the rate per cent.?
3. The interest of \$375 for 3 years and 6 months, is \$97·125. What is the rate per cent.?

4. The interest of $248 for 2 years 1 month and 20 days, is $29·194. What is the rate per cent.?

5. The interest of $184·85 for two years 8 months and 18 days, is $37·84. What is the rate per cent.?

PROBLEM 4.—Given the amount, time, and rate per cent., to find the *principal* ?

1. What principal will, in 4 years 6 months, at 8 per cent., amount to $430 ?

ART. **204.** The amount of different principals, for the same time, and at the same rate per cent., are to each other as those principals. Hence, *Dividing the given amount by the amount of* $1, *at the given rate per cent., for the given time, will give the principal.*

2. What principal will, in 7 years and 6 months, at 8 per cent., amount to $2600 ?

3. What principal will, in 2 years and 4 months, at 6 per cent., amount to $640 ?

4. What principal will, in 5 years $8\frac{4}{7}$ months, at 7 per cent., amount to $2100 ?

5. What principal will, in 4 years 4 months, at 6 per cent., amount to $3800 ?

DISCOUNT.

ART. **205.** DISCOUNT is an allowance, according to the rate per cent., made for the payment of money before it is due.

The *present worth* of a debt, payable at some future time, without interest, is such a sum as will, in the given time, and at the given rate per cent., amount to the debt. Hence, the *present worth* of any *sum* of money, payable at some future time, without interest, *is equal to the quotient arising from dividing that* SUM *by the* AMOUNT *of* $1, *at the given rate per cent., for the given time.*

The *Discount* equals the *amount*, minus the *present worth.*

1. What is the present worth of $644, due 4 years 9 months and 18 days hence, at 6 per cent.?

EXPLANATION.—\$1·288 is the amount of \$1 for the given time, and the given rate per cent. Now we have the proportion \$1·288, amount : \$644, amount : : \$1, present worth : *present worth*, required. This solved gives \$500 for the required present worth.

2. What is the present worth of \$840, due 3 years and 4 months hence, at 6 per cent.?

3. What is the present worth of \$1140, due $2\frac{1}{3}$ years hence, at 6 per cent.?

4. What is the discount on \$450, due 2 years and 9 months hence, at 7 per cent.?

5. What is the discount on \$1200, due 3 years, 4 months hence, at $4\frac{3}{4}$ per cent?

6. What is the discount on \$84·25, due 3 years 8 months and 24 days hence, at 8 per cent.?

7 What is the present worth of \$96·32, due 1 year 8 months and 12 days hence, at 6 per cent.?

8. What is the present worth of \$52·32, due 5 years 1 month 18 days hence, at 6 per cent.?

9. What is the discount on \$464·80, due 3 years 8 months and 15 days hence, at 7 per cent.?

10. Bought \$984·45 worth of goods on a credit of 9 months. How much money would discharge the debt, at the time of receiving the goods, interest being 9 per cent.?

11. A merchant bought goods to the amount of \$3328; $\frac{1}{4}$ of it was on a credit of 6 months, and the remainder on a credit of 9 months. How much money would discharge the debt, interest being $8\frac{2}{3}$ per cent.?

12. A merchant bought goods to the amount of \$2480: \$812 of which was on a credit of 3 months; \$832, on a credit of 8 months; and the remainder on a credit of 9 months. How much ready money would discharge the debt, interest being 6 per cent.?

13. A merchant bought goods to the amount of \$1600: $\frac{1}{2}$ of which was on a credit of 3 months; $\frac{1}{4}$ on a credit of 9 months; and the remainder on a credit of 1 year. How much ready money would discharge the debt, interest being 8 per cent.?

PARTIAL PAYMENTS.

ART. **206.** *Partial Payments* are payments, or indorsements,* made at various times, of a part of a *note, bond,* or *obligation.*

The method adopted by the Supreme Court of the United States for the calculation of interest on notes, and other obligations, where partial payments have been made, is as follows :—

"*Apply the payment, in the first place, to the discharge of the interest then due. If the payment exceed the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments taken together exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance, as aforesaid.*"

$860.   BETHANY, Sept. 8th, 1850.

[1.] On demand, I promise to pay Thomas Brooking, or bearer, eight hundred and sixty dollars, with interest. Value received.   JOHN JACKSON.

On this note are the following indorsements :—

Nov. 20th, 1851, received $382·24.
May 8th, 1853, " $238·45.

How much is due Dec. 29th, 1853, allowing 7 per cent. interest?

REMARK.—It will be of some assistance to the pupil to arrange the date of the note, the payments, time of settlement, and the intervals of time between payments, together with the interest of $1 for the given time, at 6 per cent., as follows :—

* Derived from a Latin phrase signifying "UPON THE BACK;" as the payments are written across the back of the note.

	years.	mo.	da.	Intervals of time. y. mo. da	Payments.	Int. of $1, at 6 per cent.
Date of note, . .	1850	9	8			
1st payment, .	1851	11	20	1 2 12	$382·24	$0·072
2d payment, . .	1853	5	8	1 5 18	238·45	0·088
Time of settlement,	1853	12	29	0 7 21		0·0385

Face of the note, or principal, . . .	$860·00
Int. on the same, at 7 per cent., to Nov. 20th, 1851,	72·24
Amount due on the note, . " " "	$932·24
First payment, ...	382·24
Amount remaining due,—2nd principal, .	$550·00
Int. on the same, from Nov. 20, 1851, to May 8, 1853,	$56·4\dot{6}$
Amount due, May 8th, 1851, . . .	$\$606·4\dot{6}$
Second payment, ...	238·45
Amount remaining due,—3rd principal, . .	$\$368·01\frac{2}{3}$
Int. from May 8th, 1853, to the time of settlement,	16·53+
Amount due Dec. 29th, 1853. . .	$\$384·54\frac{2}{3}$

$\$736\frac{00}{100}$. NEW YORK, Jan. 13th, 1848.

[2.] On demand, I promise to pay to the order of Henry Morton, seven hundred and thirty-six dollars, with interest, at 7 per cent. Value received.

RACE B. BONHOOVAN.

On this note are the following indorsements :—

Received Oct. 7th, 1849, $275·45.
" Aug. 25th, 1850, $386·38.

How much remains due Sept. 19th, 1852?

$\$684\frac{00}{100}$. CINCINNATI, July 26th, 1849.

[3.] On demand, I promise to pay James Benort, or bearer, six hundred and eighty-four dollars, with interest, at 6 per cent. Value received. JOHN P. TRUMBAL.

On this note are the following indorsements.

Received Jan. 20th, 1850, \$284·75.
" March 14th, 1851, \$84·75.
" July 26th, 1853, \$384·37½.

How much remains due Sept. 8th, 1854 ?

ANNUAL INTEREST.

In the computation of *Annual Interest*, the yearly increase of the principal is equal to the *annual* interest of the *first* principal.

If a note or obligation for \$500 should be made payable in 5 years with annual interest at 6 per cent., and another for \$500, payable in 1 year with annual interest at 6 per cent., and no payment whatever should be made on either until the expiration of 5 years, the amount of the obligations would be equal.

1. What is the annual interest of \$520 for 3 years, at 5 per cent. ?

OPERATION.

First Principal, -	\$520		
	·05		
" Interest, -	\$26·00	- -	First year's interest, \$26·00
	\$520		
	26		
Second Principal, -	\$546		
	·05		
" Interest, -	\$27·30	-	Second year's interest, \$27·30
	\$546		
	26		
Third Principal, -	\$572		
	·05		
" Interest, -	\$28·60	- -	Third year's interest, \$28·60
Annual interest of \$520 for 3 years, - - -			\$81·90

2. What is the annual interest of \$814 for 5 years, at 6 per cent. ?

3. What is the annual interest of \$840 for 6 years, at 7 per cent. ?

COMPOUND INTEREST.

COMPOUND INTEREST is interest on both principal and interest together; the interest, annually or semi-annually, being successively taken to augment the principal.

1. What is the compound interest of \$520 for 3 years, at 5 per cent.?

OPERATION.

Principal, - - - -	\$520
Per cent. - - - -	·05
Interest for 1 year, - - -	\$26·00
	520
Amount for 1 year or 2nd *principal*, -	\$546
	·05
Interest of \$546, for 1 year, - -	\$27·30
	546
Amount the 2nd year, or 3rd principal,	\$573·30
	·05
Interest of \$573·30 for 1 year, -	\$28·6650
	\$573·30
Amount the 3rd year, -	\$601·965
Original principal, - - -	\$520
Compound interest required, -	\$81·96½

2. What is the compound interest of \$384·50 for 3 years, at 8 per cent.?

3. What is the compound interest of \$840, for 2 years, interest payable semi-annually, at 8 per cent.?

4. What is the compound interest of \$460, for 3 years, interest payable half yearly, at 6 per cent.?

5. What is the difference between the annual and compound interest of \$850 for 8 years, at 6 per cent.?

BANKING AND NOTES.

ART. **209.** A BANK is an institution created by law for the purpose of issuing *bank notes*, or *bank bills*, which circulate as money, and are redeemable in specie, on presentation to the bank; also for loaning money, receiving deposits, and dealing in exchange.

The *Capital Stock* of the bank, is divided into shares

which are owned by various individuals called stockholders.

The Stockholders, annually elect a board of *Directors*, to manage the concerns of the Bank. This board elects a *Cashier*, and one of their number as *President* of the bank. The President and Cashier sign all bills issued by the Bank.

A promisory note is a positive engagement in writing, to pay a certain sum at a specified time, to a person designated in the note, or to his order, or to the bearer.

FORMS OF NOTES.

[No. 1.]

$\$78\frac{34}{100}$. LIBERTY, July 3d, 1849.

On demand, I promise to pay Edward Fox, seventy-eight and $\frac{34}{100}$ dollars, with interest. Value received.

EDWARD EVERTS.

[No. 2.]

NEGOTIABLE NOTE.

$\$99\frac{47}{100}$. KINGSTON, Aug. 25th, 1850.

One year after date, I promise to pay to the order of Moses Morton, ninety-nine and $\frac{47}{100}$ dollars, with interest. Value received. JOHN FRONTZ.

[No. 3.]

NOTE PAYABLE TO BEARER.

$\$365\frac{75}{100}$. ELLENVILLE, Sept. 10th, 1851.

Six months after date, I promise to pay Isaac Ingraham, or bearer, three hundred sixty-five and $\frac{75}{100}$ dollars, with interest. Value received. SIMEON SAWYER.

[No. 4.]

NOTE PAYABLE AT A BANK.

$\$47\frac{98}{100}$. BUFFALO, Oct. 15th, 1851

Forty days after date, I promise to pay to the order of

Joseph Langhorn, at the Union Bank, in Sullivan Co., N. Y., forty-seven and $\frac{98}{100}$ dollars, with interest. Value received.
HENRY MIFFLIN.

The *Drawer* or *Maker* of a note is the person who signs it.

Note No. 1, can be collected by Edward Fox only, therefore, it is not negotiable.

Note No. 2, becomes collectable by any person holding it, after Moses Morton writes his name on the back of it, which is called indorsing the note. Moses Morton is now called the *Indorser*. When this note becomes due payment must be demanded of the *Drawer*, and if he refuses or neglects to pay it, notice must be given without delay to the *Indorser*, demanding payment of him.

The payment of note No. 3, can be demanded by any person holding it; the *Drawer* alone is responsible.

If Joseph Langhorn writes his name on the back of note No. 4, payment may then be demanded of him, if the drawer refuses or neglects to pay it at the specified time.

A note that has not the words "*Value received*" on it, is invalid.

BANK DISCOUNT.

ART. **210.** *Bank Discount* is the sum paid to a bank for the payment of a note before it becomes due.

The amount named in a note, is called the *face* of the note. The *discount* is the interest on the face of the note for 3 days more than the time specified, and is paid in advance.* These 3 days are called *days of grace*, as the borrower is not obliged to make payment until their expiration. Hence, to compute bank discount.

Find the interest on the face of the note for 3 *days more than the* TIME *specified; this will be the discount. From the face of the note, deduct the discount, and the remainder will be the* PRESENT VALUE *of the note.*

* Taking interest in advance is usurious, and has been discontinued by many banks; and instead thereof, they deduct the true discount, found by Article 205.

1. What is the bank discount on $240 for 6 months, at 7 per cent.?

2. What is the bank discount on $460 for 4 months, at 8 per cent.?

3. What is the bank discount on $150·50 for 2 months and 15 days, at 6 per cent.?

4. What is the bank discount on $475·85 for 3 months, at 7 per cent.?

5. What is the bank discount of a note of $875·50, for 3 months 21 days, at 7 per cent.?

6. What sum must a bank pay for a note of $385·75, payable in 6 months, discount, at 7 per cent.?

7. What is the present value of a note of $875·25, discounted at a bank for 8 months and 9 days, at 6 per cent.?

8. What is the present value of a note of $846·50, discounted at a bank for 4 months and 15 days, at 7 per cent.?

9. What is the present value of a note of $8484·50 discounted at a bank for 7 months and 9 days at 7 per cent.?

ART. **211.** Given the *present value* of a bankable note, the rate per cent., and the time for which it is to be discounted, to find the face of the note.

1. What must be the face of a bankable note so that when discounted for 4 months and 15 days, at 6 per cent., it shall give a present value of $1954?

SOLUTION.—The discount of $1 for the given time, at the given rate per cent., is $·023; hence, $1—$·023=$·977, the the present value of $1 for the given time, at the given rate per cent.

We now have the proportion, $·977, present value : $1954, present value : : $1 amount, : *required amount,* which is $2000, the face of the note.

Hence, we infer in general, that if we *Divide the given* PRESENT VALUE *by the* PRESENT VALUE *of* $1 *for the given time and at the given rate per cent., bank discount; the quotient will be the amount or face of the note.*

2. What must be the face of a bankable note, so that when discounted for 6 months and 10 days, at 6 per cent., it will give $85, present value?

3. What must be the amount of a bankable note, so that when discounted for 4 months and 21 days, at 7 per cent, it shall give \$84·95 present value?

4. What must be the amount of a bankable note so that when discounted for 4 months and 9 days, at 8 per cent., the borrower shall receive \$384?

5. What must be the amount of a bankable note, so that when discounted for 6 months and 27 days, at 7 per cent., the borrower shall receive \$580?

AVERAGE.

ART. **212.** If the sum of a series of *promiscuous* quantities, be divided by the number of quantities, the quotient will be one of a series of *equal* quantities, whose sum will equal the sum of the former series. This quotient is called the AVERAGE of the given quantities.

1. What is the average of 12, 16, and 20?

2. During six successive months a laborer saved \$12, \$18, \$25, \$30, \$20, and \$27 a month respectively. How many dollars did he average a month?

3. A locomotive made 4 successive trips over a track 20 miles in length, in the following times: 30 minutes 25 seconds; 25 minutes 15 seconds; 33 minutes 10 seconds; and 24 minutes 30 seconds. What was the average time of 1 trip, also of running 1 mile?

MERCANTILE CALCULATIONS.

EQUATION OF PAYMENTS.

ART. **213.** EQUATION OF PAYMENTS is the process of finding the average time for the payment of several sums due at different times, without loss to either party.

The rules applying to *mercantile calculations* will be given for the accommodation of book-keepers.

ART. **214.** The equated time for the payment of any sum, when parts of it are payable at different times, may be found by the following

RULE.

Multiply each payment by the time that must elapse before it becomes due; then divide the sum of these products by the sum of the payments. The quotient will be the average time required.

1. I purchased goods to the amount of $1200; $300 of which I am to pay in 2 months; $400 in 3 months; and $500 in 6 months. How long a credit ought I to receive, if I pay the whole at once?

OPERATION.

$	mo.	mo.	
300	× 2 =	600	EXPLANATION.—A credit on $300 for 2 months is the same as the credit on $1 for 600 months.
400	× 3 =	1200	A credit on $400 for 3 months is the same as the credit on $1 for 1200 months.
500	× 6 =	3000	A credit on $500 for 6 months is the same as the credit on $1 for 3000 months.
1200)		4800(4 mo.	Therefore, on the whole sum, $1200, I should receive the same as the credit, or the *interest* on $1 for 4800 months; the $1200 will give the same interest in one-twelve hundredth of 4800 months =4 months, the time in which the whole amount averages due.
		4800	
		0	

2. If I owe $900; $200 of which is due in 2 months; $300 in 4 months; and the remainder in 6 months. What is the average time for the payment of the whole?

3. If you owe a man $150, payable in 2 months; $260 payable in 4 months; $490 payable in 8 months; at what time may you in equity pay the whole?

4. A merchant bought goods to the amount of $400, on a credit of 4 months; another quantity for $500, on a credit of 5 mo.; and another quantity for $800, on a credit of 8 mo. What is the average time for the payment of the whole?

5. A gentleman owes a certain sum of money; $\frac{1}{3}$ of which is due in 3 months; $\frac{1}{2}$ in 4 months; $\frac{1}{6}$ in 12 months. What is the average time of payment?

REMARK.—We will suppose the amount owed is $1, as it can make no difference what that amount is, since certain fractional parts of it become due at particular times.

6. A merchant bought goods to the amount of $3000;

$\frac{2}{5}$ of which he paid in cash at the time of receiving the goods; $\frac{1}{3}$ is to be paid in 6 months; and the remainder in 1 year and 3 months. What is the average time for the payment of the whole?

7. A man purchased a farm for \$3200, and agreed to pay \$500 of it at the expiration of 3 months; \$1200 at the expiration of 9 months; and the remainder in 12 months. What is the equated time for the payment of the whole?

ART. **215.** In *mercantile* transactions it is customary to give a credit of from 3 to 9 months, on bills of sale.

ART. **216.** To determine the average time of payment of several sales, on different terms of credit, we have the following

RULE.

Multiply the amount of each sale by the time intervening between the date on which the first amount falls due, and the date on which each sum falls due. Then divide the sum of these products by the whole amount of debt, and the quotient will be the averaged time of payment, to be counted forward from the date of the first amount falling due.

1. Purchased goods of Stilwell, Brown & Co., at different dates and on different terms of credit; as below stated.

Feb.	2, 1853, a	bill	amounting to	\$460 on 3	months'	credit.
Feb.	5, "	"	"	\$680 on 4	"	"
March	28, "	"	"	\$560 on 5	"	"
April	12, "	"	"	\$840 on 5	"	"

I wish to make one payment of the whole debt. When, per average, will it become due?

SOLUTION.—The above bills come due respectively as follows:

				days.		days.
Due May	2,	\$460	×	00	=	00000
" June	5,	\$680	×	34	=	23120
" Aug.	28,	\$560	×	118	=	66080
" Sept.	12,	\$840	×	133	=	111720
		\$2540)				200920(79 days.
						&c. &c.

The $460 become due May 2nd, 1853; the $680 become due 34 days from May 2nd; the $560, 118 days from May 2nd; and the $840, 133 days from May 2nd. By equation of payments I find these bills will average due in 79 days from May 2nd, which is July 20th, 1853.

☞ If it were required to know how much money would balance the account any time previous to July 20th, as April 15th, it is evident that the *present worth* of $2540 from April 15th to July 20th, would be the sum required.

When the different sales are made on the same terms of credit, the average time for the payment of the whole debt, may be found as taught by the following question:

2. A merchant sold goods to one of his customers, at different dates; as below stated:

April 8, 1853,	a bill amounting to	$470 on 6	month's credit.
May 17,	" " "	$840 on 6	" "
June 23,	" " "	$980 on 6	" "
July 10,	" " "	$580 on 6	" "

What is the average time for the payment of the above bills?

OPERATION.

		days.	days.
April 8, 1853,	$470 ×	00 =	00000
May 17, "	$840 ×	39 =	32760
June 23, "	$980 ×	76 =	74480
July 10, "	$580 ×	93 =	53940
	$2870)		161180(56 days.

From a little reflection the pupil will discover that the above bills will average due in 56 days from the time the first falls due, which is Dec. 3, 1853.

3. A merchant sold to one of his customers several parcels of goods, at sundry times, and on different terms of credit; as follows:

Feb. 1,	a bill amounting to	$300 on 4	months' credit.
March 7,	" "	$185 on 5	" "
April 15,	" "	$280 on 4	" "
May 20,	" "	$210 on 3	" "

What is the equated time for the payment of all these bills?

4. Purchased goods of a merchant at different times, and on different terms of credit as follows:

March 3, 1853, a bill amounting to	$847·10	on 3	months'	credit.
April 5, " " "	$645.50	on 4	"	"
May 10, " " "	$584·75	on 6	"	"
June 15 " " "	$475·84	on 8	"	"
Aug. 18 " " "	$384·95	on 4	"	"

What is the equated time for the payment of the above bills?

5. Purchased goods at sundry times, and on different terms of credit, as follows:

June 4, 1853, a bill amounting to	$485·90	on 3	month's	credit.
July 12, " " "	$675·25	on 4	"	"
Aug. 15, " " "	$812·12½	on 5	"	"
Sept. 22, " " "	$895·25	on 6	"	"
Nov. 20, " " "	$896·70	on 4	"	"

What is the equated time for the payment of all these bills?

6. Bought goods of J. B. Smith & Co., at sundry times, as shown by the statement annexed.

March 2, 1853, a bill amounting to	$684·20	on 6	months'	credit.
March 28, " " "	$375·54	"	"	"
April 10, " " "	$484·40	"	"	"
May 20, " " "	$795·45	"	"	"
June 30, " " "	$840·60	"	"	"

How much money will balance the account, July 4, 1854?

7. Bought goods of C. B. Hill & Co., at different times, and on different terms of credit, as shown by the statement annexed?

June 12, 1853, a bill amounting to	$340·65	on 5	months'	credit.
July 8, " " "	$595·75	"	"	"
Aug. 10, " " "	$784·85	6	"	"
Oct. 14, " " "	$987·90	8	"	"
Nov. 15, " " "	$878·98	4	"	"
Dec. 19, " " "	$999·99	2	"	"

How much money will balance the account, Jan. 20, 1854?

8. Bought goods of R. Lancaster & Co. as shown by the statement annexed;

May 4, 1853,	a bill	amounting	to	$432·95	on 3	months'	credit.
May 18, "	"	"		$843·45	2	"	"
Jnne 20, "	"	"		$732·46	5	"	"
July 8, "	"	"		$846·75	7	"	"
Aug. 20, "	"	"		$784·78	6	"	"
Sep. 24, "	"	"		$976·34	4	"	"

What is the equated time for the payment of all these bills; and how much money would balance the account, Nov. 12, 1854?

ART. **217.** The rule given for the Equation of Payments is the one usually adopted by merchants, although not strictly correct, still it is sufficiently accurate for all practical purposes, when small sums and short periods of time are considered.

This inaccuracy will become evident by inspecting the following example:—

A owes B $4480; $2240 of which is due in 2 years, and the remainder in 10 years. What is the equated time for the payment of the whole, interest 6 per cent.?

The average time of payment found by the rule above referred to, is 6 years. From which we observe that $2240 is not paid until 4 years after it is due, also that $2240 is paid 4 years before it is due;—these two conditions are considered to mutually counter-balance each other, although, they do not. It is evident, that, for deferring the payment of the first $2240 for 4 years, A should pay the *amount* of $2240 for the same time, which is $2777·60; but for the remainder, which he pays 4 years before it is due, he should pay the present worth of $2240 for the same time, which is $1806·45. Hence the RULE occasions an error of $2777·60+$1806·45—$4480=$104·05.

Justice demands that interest should be required on all sums from the time they become due until the payment is made, and the *present worth* of all sums paid before they become due. Hence the following accurate

RULE.

Find the present worth of each of the given amounts due, then find in what time the sum of these present worths will amount to the sum of all the payments.

ART. **218.** When a debt due at some future period has received partial payments before the time due, to find how long after this time, the remainder may in equity remain unpaid.

RULE.

Divide the sum of the products of each payment into the time it was paid before due, by the sum remaining unpaid The quotient will be the required time.

1. A owes $1200, due in 6 months; five months before it is due $200 is paid; and 3 months before it is due, $300 is paid. How long after the expiration of the 6 months may the remaining $500 in equity remain unpaid?

OPERATION.

	mo.		mo.
$200 ×	5	=	1000
$500 ×	3	=	1500
$500)			2500
			5 mo.

EXPLANATION.—A credit on $200 for 5 months is the same as a credit on $1 for 1000 months.

A credit on $500 for 3 months is the same as a credit on $1 for 1500 months.

The money paid in advance affords a profit equal to the interest of $1 for 2500; the balance remaining due, $500, will afford the same interest in one-five hundredths of 2500 months, which is 5 months.

2. A person owes $400, due at the end of 10 months. At the end of 4 months he pays $100; 3 months after that he pays $50. How long after the expiration of the 10 months may the balance remain unpaid?

3. A merchant lends to a farmer $1600, payable in 12 months. At the end of 4 months $200 of it is paid; 3 months after that $400 more is paid; and 1 month before the expiration of the 12 months $200 more is paid. How long after the expiration of the 12 months may the balance in equity remain unpaid?

4. A lends B $1200 for 6 months; at another time $1800 for 8 months. For how long a time ought B to lend A $2700, to balance the favor?

5. I borrowed of my neighbor $900 for 5 months; at another time $800 for 9 months. For how long a time ought I to lend my neighbor $850 to balance the favor?

COMPOUND EQUATION OF PAYMENTS.

ART. **219.** *Compound Equation* of Payments teaches the method of ascertaining the time on which the balance of an account that contains debit and credit becomes due; having first learned, by Rule under Art. 216, when the debit and credit of said account falls due, respectively, without regard to their relation to each other.

RULE.

1st. *Multiply the number of days between the dates of* EQUATED TIME *by the amount that first falls due; and divide this product by the difference between the debit and credit of the account; the quotient will be the* TIME *for consideration.*

2d. *If the larger amount comes due first, the* TIME *is counted* BACK *from the* LATEST *date; but if the smaller amount comes due first, the* TIME *is counted* FORWARD *from the* LATEST *date.*

1. By equation of payments it is found that A's account with B. is as follows:

	Dr.		*Cr.*
Due June 4th, . .	$400	Due July 24th, . .	$900

When will the balance of the account become due?

OPERATION

Amount of Cr. $900 due July 24th.
" Dr. $400 due June 4th.

Balance $500 From June 4th to July 24th, is 50 days.

days.
50
400

500)20000

40 days from July 24th, which is Sept. 2d, the balance becomes due.

EXPLANATION—It is evident that B should receive the interest on $400 from June 4th to July 24th. Therefore B should retain

the balance ($500) sufficiently long after it becomes due, to receive the same amount of interest on it as he would have received on the $400 from June 4th to July 24th.

If $400 in 50 days give a certain interest, $1 will give the same interest in 400 times 50 days = 20000 days; and $500 (the balance) will give the same interest in $\frac{1}{500}$ of 20000 days =40 days; consequently, in 40 days from July 24th, the balance becomes due.

2. Suppose the above account to stand as follows?

	Dr.		Cr.
Due June 4, . . .	$900	Due July 24, . . .	$400

At what time must a note for the balance be dated, to balance the account?

SOLUTION.—It is evident that B should receive the interest on $900 from June 4th to July 24th. Therefore, to balance the account, B should receive a note of $500 (the balance), of such a date that the interest on it should, on the 24th day of July, equal the interest on $900 for 50 days, the time from June 4th to July 24th.

If $900 in 50 days give a certain interest, $1 will give the same interest in 900 times 50 days = 45000 days; and $500 will give the same amount of interest in $\frac{1}{500}$ of 45000 days. Hence, 90 days previous to July 24th, which is April 25th, the note should be dated.

3. B has with C the following account :—

1853.	Dr.	1854.	Cr.
Nov. 12. Due . .	$840.	Jan. 20. Due . .	$1280.

When will the balance of the account become due?

4. C has with D the following account :—

1853.	Dr.	1853.	Cr.
July 20. Due . .	$987.	Sept. 4. Due . .	$507

At what time must a note of the balance be dated to balance the account?

5. At what time will the balance of the following account become due?

1853.	Dr.	1854.	Cr.
Oct. 26. Due . .	$1280.	Jan. 16. Due . .	$840.

6. When will the balance of the following account become due?

1853.	*Dr.*	1853.	*Cr.*
April 21. Due . .	$845	June 15. Due . .	$1685

Cash Balance.

Art. **220.** To find the cash balance of an account consisting of various items of debit and credit, of different dates, at any specified time.

RULE.

Place on the debtor or credit side, such a sum, (which may be called MERCHANDISE BALANCE,*) as will balance the account. Multiply the number of dollars in each entry by the number of days from the time the entry was made to the time of settlement; and the merchandise balance by the number of days for which credit was given. Then multiply the difference between the sum of the debit, and the sum of the credit products by the interest of* $1 *for* 1 *day; this product will be the* INTEREST BALANCE.

When the sum of the debit products exceed the sum of the redit products, the interest balance is in favor of the debit side; but when the sum of the credit products exceed the sum of the debit products, it is in favor of the credit side. Now to the merchandise balance add the interest balance, or subtract it, as the case may require, and you obtain the CASH BALANCE.

1. A has with B the following account :—

1849.		*Dr.*	1849.		*Cr.*
Jan. 2.	To merchandise	$200	Feb. 20.	By merchandise	$100
April 20.	" "	400	May 10.	" "	300

If interest is estimated at 7 per cent., and a credit of 60 days is allowed on the different sums, what is the cash balance August 20, 1849?

Explanation.—Without interest, the cash balance would be $200.

If no credit had been given, the debits should be increased by the interest of $200 for 230 days, at 7 per cent.; and the interest of $400 for 122 days, at 7 per cent. The credits should be increased by the interest of $100 for 181 day, at 7 per cent.; and the interest of $300 for 102 days, at 7 per cent.

Since a credit of 60 days is given on all sums, it is evident

by the above calculation, that we shall increase the debits by the interest of the sum of the debits, \$600, for 60 days more than justice requires. Also, that we should increase the credits by the interest of the sum of the credits, \$400, for 60 days more than we should do.

Now, instead of deducting these items of interest from the *amount* of debit and credit interests, it is plain, that it will be more convenient and equally just, to diminish the debit interest by the interest of the *merchandise balance* for 60 days, which can be most readily accomplished by adding the interest on the merchandise balance for 60 days, to the credit items of interest.

From which we discover that the *interest balance* is equal to the difference between the sum of the debit interests, and the sum of the credit interests increased by the interest of the merchandise balance for the time for which credit was given.

OPERATION.

DEBITS.		CREDITS.
\$ Days.		\$ Days.
$200 \times 230 = 46000$		$100 \times 181 = 18100$
$400 \times 122 = 48800$		$300 \times 102 = 30600$
94800	Balance,	$200 \times 60 = 12000$
60700		60700

$$\frac{0{\cdot}07}{365} \times 34100 = \$6{\cdot}54 \textit{ Interest balance, nearly.}$$

Therefore, the foregoing account becomes balanced as follows :—

1849.		Dr.	1849.		Cr.
Jan. 2.	To merchandise,	\$200·00	Feb. 20.	By merchandise,	\$100·00
April 20.	" "	400 00	May 10.	" "	300·00
Aug. 20.	" balance of interest,	6·54	Aug. 20.	" balance,	206·54
		\$606·54			\$606·54
Aug. 20.	" Cash balance,	\$206·54			

NOTE.—It is customary in practice, when the number of cents in any of the entries, are less than 50, to omit them, and to add \$1 when they are 50 or more.

2. A has with B the following account :—

1852.		Dr.	1852.		Cr.
Jan. 8.	To merchandise,	\$400	Feb. 10.	By merchandise,	\$300
April 24.	" "	800	May 24.	" "	500

If interest is estimated, at 7 per cent. and a credit of 60 days is allowed on the different sums, what is the cash balance Sept. 25th, 1852?

3. B has with C the following account :—

1853.		*Dr.*	1853.		*Cr.*
Feb. 12.	To merchandise,	\$840	March 16.	By merchandise,	\$640
July 25.	" "	980	May 14.	" "	780
Aug. 14.	" "	640	Sept. 20.	" "	430

If interest is estimated at 8 per cent., and a credit of 90 days is allowed on the different sums, what is the cash balance Jan. 10th, 1854?

TRADE AND BARTER.

ART. **221.** *Trade and Barter* is the exchange of one commodity for another without loss to either party.

1. How many yards of muslin, at \$·12½ a yard, must be given for 380 lbs. of butter, at \$·16 a pound?

2. How many bushels of rye, at \$·93¾ a bushel, must be given for 187½ lbs. of tea, at \$·62½ a pound?

3. A merchant exchanged 630 yards of cloth, for 15 hogsheads of wine, at \$1·10 a gallon. How much was the cloth a yard?

4. A farmer gave 20½ cwt. of hops, at \$6·80 per cwt., for 8 cwt. 3 qrs. 20 lbs. of sugar, and \$80 in money; at how much was the sugar valued per pound?

5. A farmer received for 25 cwt. 2 qrs. 22 pounds of cheese, at \$·08½ a pound, 18 yards of cloth, at \$2·50 a yard; 16 yards of muslin, at 5½ cents a yard; 5 pair of boots, at \$2·75 a pair; 85 gallons of molasses, at \$·18¾ a gallon, and the balance in sugar, at \$·09½ per pound. How many pounds of sugar did he buy?

6. A grocer barters 860 bushels of oats, which cost him \$·25 a bushel, at \$·37½ a bushel, for cloth that cost \$2·86⅔ a yard. What is the bartering price of the cloth, and how many yards did the grocer receive?

7. A farmer has 184½ bushels of rye, which is worth \$·84 per bushel; but in barter he is willing to put it at

\$·56 a bushel, providing his neighbor will let him have wheat worth \$1·24 per bushel for \$·91. Will he gain or lose by the bargain, and what per cent.?

8. Two farmers bartered: A had 240 bushels of wheat, at \$1·50 per bushel, for which B gave him 200 bush. of corn, at \$·65 per bush., and the balance in buckwheat, at \$·80 a bushel. How much buckwheat did A receive of B?

9. A farmer has 380 bushels of wheat, worth \$1·20 a bushel; but in barter he will have \$1·44 a bushel. A merchant has broadcloth worth \$3·60 a yard; and linen worth \$1·40; at what price per yard ought the merchant to rate his broadcloth and linen to be equivalent to the farmer's bartering price, and how many yards will the farmer receive for his wheat, providing he takes an equal number of yards of each?

10. A and B barter: A has 560 bushels of wheat worth \$1·20, but in barter he will have \$1·60 a bushel; B has broadcloth worth \$4·20 a yard; how must B sell his broadcloth a yard in proportion to A's bartering price for his wheat, and how many yards are equal in value to A's wheat?

11. A had 450 yds. of cloth, worth \$1·20 a yard, which he bartered with B, at \$1·45 a yard; taking flour, at \$7·50 a barrel, which is worth but \$6. How much flour will pay for the cloth; and who gets the best of the bargain?

12. A farmer sold to a merchant one yoke of oxen for \$125; 184 bushels of corn, at \$·37½ a bushel; 45 bushels of wheat, at \$·93¾ a bushel, April 14th, 1853. In payment he received 125 lbs. of raisins, at 10¾ cents a pound; 584 pounds of sugar, at 9½ cents a pound; and 54 gallons of molasses, at 13⅔ cents a gallon, Nov. 8th, 1853 How much remains due; interest 6 per cent.?

13. A farmer took to market 2670 lbs. of wheat, worth \$·93¾ a bushel; and in payment takes \$5·17⅞ to pay his taxes; and for the remainder he is to receive an equal number of yards of muslin, at 7¼ cents a yard; bleached muslin, at 12½ cents; calico, at 16⅔ cents; and linen, at 31¼ cents. How many yards of each did he buy?

14. A farmer, Mr. Smith, lent his neighbor, April 1st, 16 bushels of superior wheat, on condition that it should be paid in wheat of equal quality, on the first of the following November, after adding 3 per cent. to it for its use. The wheat his neighbor returned was 7 per cent. inferior to that which he received. How many bushels of wheat should Mr. Smith in equity receive?

15. A farmer, Mr. Jackson, owes a merchant $560, May 1st, 1853. On the above, Mr. J. paid, Aug. 4th, 1853, $4\frac{3}{4}$ bushels of clover-seed, at $6·75 a bushel, and a yoke of oxen for $95. On Nov. 15th, 1853, Mr. J. paid $63\frac{1}{2}$ bushels of rye, at $·$93\frac{3}{4}$ a bushel, and the remainder in wheat, at $1·$12\frac{1}{2}$ a bushel. How many bushels did it take, interest 8 per cent.?

16. A Mr. Judson sold to Mr. Wilson, April 10, 1852; 4 cows, at $23·75 each; 15 bushels of oats, at $·$37\frac{1}{2}$ a bushel; 24 cwt. 3 qrs. of hay, at $10·75 a ton; and 1 wagon, at $84·95. Mr. Wilson, in payment, sold Mr. J., March 15th, 1853, 3 plows, at $6·$37\frac{1}{2}$ each; 2 cultivators, each $5·$18\frac{3}{4}$; $12\frac{1}{2}$ yards of broadcloth, at $4·85 per yard; 2 barrels of sugar, at $17·75 a barrel; 5 sacks of salt, at $3·$12\frac{1}{2}$ a sack. Allowing 7 per cent. interest, how does the account stand, Oct. 12th, 1853.

17. A speculator, Mr. Manning, bought of Mr. Bronson a house and lot for $2400, January 1st, 1853, $\frac{1}{4}$ of which was payable at the time the purchase was made, and the remainder was to be paid, with interest at 7 per cent. in 3 equal payments; the first in 4 months; the second in 8 months; and the third in 12 months. Mr. B. sold Mr. M. $485\frac{1}{2}$ bushels of potatoes, at $·$62\frac{1}{2}$ a bushel, April 15th, 1853; and August 12th, 1853, 748 bushels of corn, at $·$47\frac{1}{2}$ a bushel. They are desirous of settling, Nov. 18th, 1853. How much in equity should Mr. Manning pay Mr. Bronson?

18. Mr. Mathews sold to Messrs. Arnold & Co., May 12, 1853, 465 lbs. of pork, at $·$11\frac{1}{2}$ a pound, and a span of horses and pleasure wagon for $684·50, on 6 months' credit. June 15, 1853, Mr. M. bought of Messrs. A. & Co. 47·75 acres of land, at $23·25 an acre, on 3 months' credit. Mr

M. sold to Messrs. A. & Co. 184 bushels of wheat, at $\$ \cdot 93\frac{3}{4}$ a bushel, for which no credit is allowed. They settled accounts Nov. 15th, 1853. Which was in debt to the other, and how much ?

19. A farmer sold to a merchant $47\frac{1}{2}$ bushels of corn, at $\$ \cdot 65$ a bushel ; $84\frac{1}{2}$ bushels of rye, at $\$ \cdot 87\frac{1}{2}$ a bushel ; $36\frac{3}{4}$ bushels of buckwheat, at $\$ \cdot 93\frac{3}{4}$ a bushel ; and in payment received $12\frac{1}{2}$ lbs. of tea, at $\$1 \cdot 12\frac{1}{2}$ a pound ; $15\frac{1}{2}$ pounds of coffee, at $\$ \cdot 16\frac{2}{3}$ a pound ; 135 pounds of sugar, at $\$ \cdot 9\frac{3}{4}$ a pound ; 25 gallons of molasses, at $35\frac{1}{2}$ a gallon ; $36\frac{1}{2}$ yards of linen, at $\$ \cdot 15\frac{1}{2}$ a yard; $25\frac{1}{2}$ yards of calico, at $\$ \cdot 16\frac{2}{3}$ a yard ; $24\frac{1}{2}$ yards of broadcloth, at $\$3 \cdot 85\frac{1}{2}$ a yard ; 15 pair of shoes, at $2·54 a pair ; and a set of spoons, knives, and forks, for $19·84. Which owed the other, and how much ?

20. A farmer sold a mechanic, April 1st, 1853, a span of horses for $284; a yoke of oxen for $184; $148\frac{1}{2}$ bushels of grain, at $\$ \cdot 93\frac{3}{4}$ a bushel; and 4 cows, each $25·75, on a credit of 9 months. The mechanic sold the farmer, May 1, 1853, a lumber wagon for $184; a pleasure wagon for $325; and 4 plows, each $6·75, on a credit of 3 months. They settle accounts Sept. 1st, 1853; which is in debt, and how much, interest 6 per cent. ?

21. A farmer sold to a merchant, Jan. 3d, 1852, 1764 pounds of pork, at $8\frac{2}{3}$ cents a pound; 1683 pounds of beef, $4\frac{3}{4}$ cents a pound; 847 pounds of ham, at $10\frac{1}{2}$ cts. a pound; March 12th, 1852, 485 bushels of oats, at $\$ \cdot 43\frac{1}{2}$ a bushel; 184 bushels of rye, at $\$ \cdot 62\frac{1}{2}$ a bushel; 284 bushels of wheat, at $\$ \cdot 93\frac{3}{4}$ a bushel; and 487 pounds of cheese, at $11\frac{1}{2}$ cents a pound. The farmer received of the merchant, March 25th, 1853, merchandise to the amount of $684·75; June 15th, 1853, merchandise to the amount of $846·45. In the above transaction 6 months' credit was given on all the articles. Balance the account July 3d, 1853, at 6 per cent. interest.

22. Mr. Smith bought of a speculator a farm containing 372 acres, at $40 an acre, and was to pay for it in ten years as follows : $\frac{1}{8}$ of the whole the first year; $\frac{1}{7}$ of the remainder the second year; $\frac{1}{6}$ of the remainder the third year;

$\frac{1}{5}$ of the remainder the fourth year; $\frac{1}{4}$ of the remainder the fifth year; $\frac{1}{3}$ of the remainder the sixth year; and the remainder in four equal and annual payments, without interest. The first payment was made in wheat, at \$1·12$\frac{1}{2}$ a bushel; the second in wheat, at \$1·25 a bushel; the third in wheat, at \$·93$\frac{3}{4}$ a bushel; the fourth in wheat, at \$·95 a bushel; the fifth in wheat, at \$1·10 a bushel; the sixth in wheat, at \$1·20 a bushel; and the remainder in wheat, at \$1·15 a bushel. What was the amount of each payment, and the number of bushels of wheat paid annually? If no payment had been made until the end of the ten years, how many bushels of wheat, at \$1·35 a bushel, would have balanced the account, interest, at 7 per cent.?

CHAPTER IX.

PROGRESSION.

ARITHMETICAL PROGRESSION.

ART. **222.** A SERIES of numbers that increase or decrease by a constant difference, is said to be in Arithmetical Progression. When the terms are constantly increasing, the series is called an ASCENDING *Arithmetical Progression;* as, 1, 3, 5, 7, 9, 11, 13, 15, &c.

When they are constantly decreasing, the series is called a DESCENDING *Arithmetical Progression;* as,

45, 43, 41, 39, 37, 35, 33, 31, &c.

The first and last terms of a Progression are called the *extremes*, and the other terms are called the *means*.

From the nature of an Arithmetical Progression, it follows, that the sum of the extremes is equal to the sum of any other two terms equally distant from them, or to *twice* the middle term; if the number of terms is unequal. This

will appear more plain by inspecting the following Progression :—1, 3, 5, 7, 9, 11, 13, 15, and 17.

1, 3, 5, 7, 9 17, 15, 13, 11, 9 — — — — — 18, 18, 18, 18, 18	Here the sum of the extremes and the terms equally distant from them, are added, and found to be equal, as above stated.

In Arithmetical Progression there are FIVE distinct terms to be considered :—

a, The first term;
l, The last term;
n, The number of terms;
d, The common difference; and
s, The sum of all the terms.

These terms are so related that any *three* of them being known, the remaining two may be found. Since there are *five* terms, and only *three* of them necessary to be known, to find a fourth, it follows that there may be TWENTY distinct *cases* in Arithmetical Progression. We shall, however, notice but few of them, and refer the student to Algebra for the others.

CASE I.

ART. **223.** Given the first term, the common difference, and the number of terms, to find the last term.

1. What is the 25th term of an arithmetical progression, the first term of which is 6 and the common difference 4 ?

EXPLANATION.—The second term of an arithmetical progression ascending is equal to the first term *plus* the *common* difference; the *third* term is equal to the first term *plus twice* the common difference; and so on. Therefore, when we have given the first *term*, the *common* difference and the number of terms, the *last* term is found by *Adding the first term to the product of the common difference into the number of terms less one.*

2. A man bought 25 acres of land, giving $2 for the first acre, $8 for the second, $14 for the third, and so on,

increasing in arithmetical progression. What did the last acre cost at this rate?

3. A merchant bought 18 pieces of cloth, giving \$3 for the first piece; \$5 for the second; \$7 for third, and so on, increasing in arithmetical progression. What, at this rate, did the last piece cost?

4. A tapering board, $3\frac{1}{2}$ inches wide at the narrow end, and 14 feet long, is found to increase in width $1\frac{2}{3}$ inches for every foot in length. What is the width of the wide end?

5. In a certain orchard there are 34 rows; in the first row there are 20 trees; in the second 24; in the third 28; and so on, the number of trees in each row continuing to increase in an arithmetical ratio. How many trees, at this rate, are there in the last row?

CASE II.

ART. **224.** Given the first term, the last term, and the number of terms, to find the sum of all the terms.

1. The first term of an arithmetical progression is 5, the last term is 85, and the number of terms is 12. What is the sum of all the terms?

EXPLANATION.—The sum of the extremes of an arithmetical progression being equal to the sum of any two terms equally distant from them, it follows that the terms must average half the sum of the extremes; hence, the *Sum of all the terms equals the product of the number of terms by half the sum of the extremes.*

2. A man bought 25 acres of land: for the first acre he gave $\$\frac{1}{2}$; for the last, $\$244\frac{1}{2}$; the prices of the successive acres form an arithmetical series. How much did the 25 acres cost at this rate?

3. A merchant bought 25 pieces of cloth; for the first piece he gave \$3; for the last piece, \$63; the prices of the pieces form an arithmetical progression. How much at this rate, did the cloth cost him?

4. In a certain field there are 44 rows of corn: in the first row there are 10 hills; and in the last, 139 hills; the number of hills in the successive rows form an arith-

metical progression. How many hills are there in the field ?

CASE III.

ART. **225.** Given the extremes and the common difference, to find the number of terms.

1. The first term of an arithmetical progression is 8; the last term 83 ; and the common difference 5. What is the number of terms ?

EXPLANATION.—Since the last term of an arithmetical progression equals, the product of the number of terms less one into the common difference, increased by the first term, (see Case 1;) it follows *that the number of terms equals the quotient, increased by* 1, *arising from dividing the difference of the extremes by the common difference.*

2. A man going a journey traveled the first day 7 miles, the last day 67 miles, and each day increased his journey by 4 miles. How many days did he travel ?

3. A merchant bought a certain number of pieces of cloth, the prices of which increased by $2. The first piece cost $3, and the last piece $43. How many pieces did he buy ?

GEOMETRICAL PROGRESSION.

ART. **226.** A SERIES of numbers that increase or decrease by a constant *multiplier*, is said to be in *Geometrical Progression.*

When the constant *multiplier*, which is called the RATIO, is greater than a unit, the series is called an *Ascending Geometrical Progression ;* as,

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, &c.

When the ratio is less than a unit, the series is called a *Descending Geometrical Progression ;* as,

512, 256, 128, 64, 32, 16, 8, &c.; or,

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$, &c.

In Geometrical, as in Arithmetical Progression, there are FIVE terms to be considered :—

a, The first term;
l, The last term;
n, The number of terms;
r, The ratio; and
s, The sum of all the terms.

These terms are so related that any *three* of them being known, the remaining two may be found. Since there are *five* terms, and only *three* of them necessary to be known, to find a fourth; it follows that there may be TWENTY distinct *cases* in Geometrical Progression. We shall, however, notice but two of them, and refer the student to Algebra for the others.

CASE I.

ART. **227.** Given the first term, the number of terms, and the ratio, to find the last term.

1. The first term of a geometrical progression is 2, the ratio is 3, and the number of terms is 8. What is the last term?

SOLUTION.—From the nature of geometrical progression, it is evident, that the *second* term equals the *first* term, multiplied by the *ratio;* the *third* term equals the first, multiplied by the ratio *squared;* the *fourth* term equals the *first*, multiplied by the ratio *cubed,* and so on for the following terms; Hence, *the first term multiplied by that power of the ratio denoted by the number of terms, less one, will give the last term.*

2. A person traveling, goes 3 miles the first day, 6 miles the second day, 12 miles the third day, and so on, increasing in geometrical progression, for 6 days. How far did he go the last day?

3. An individual commenced business with a capital of $20, and was so fortunate as to double it once in every two years; what was his capital, at the end of 25 years?

CASE II.

ART. **228.** Given the first term, the last term, and the ratio, to find the sum of all the terms.

1. The first term of a geometrical progression is 4, the last term is 12500, and the ratio is 5. What is the sum of all the terms?

EXPLANATION.—If from *ratio* times any series, as 2, 8, and 32, we subtract the *series*, the remainder will be RATIO times 32—2. It is also evident, that if from *ratio* times the *series*, we subtract, *once* the *series* there will remain, (RATIO—1) times the *series*, which must equal *ratio* times 32—2. Therefore, the sum of the series equals 4 times 32—2÷(4—1.) Hence, *Multiply the last term by the ratio; from the product subtract the first term and divide the remainder by the ratio diminished by one,* and it will give the sum of all the terms.

2. A gentleman engaged a horse and carriage to ride 10 miles, and agreed to pay $\frac{1}{2}$ of a cent for the first mile; 1 cent for the second; two for the third; and so on, increasing in geometrical progression. How much at this rate, did the 10 miles' ride cost him?

3. A speculator sold 10 horses on this condition: that he should pay \$3 for the first horse; \$9 for the second; \$27 for the third, and so on, increasing in geometrical ratio. What did the last horse cost, and what did they all cost?

SUMMATION OF AN INFINITE DECREASING SERIES. An Infinite Series is one that, being continued, would run on *ad infinitum.* If a decreasing geometrical series, as 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, &c., be continued to *infinity* the last term evidently may be considered 0. The sum of such a series may be determined as follows.

Divide the first term by a unit diminished by the ratio.

1. What is the sum of the infinite series, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c.?
2. What is the sum of the infinite series, 1, $\frac{1}{3}$, $\frac{1}{9}$, &c.?
3. What is the sum of the infinite series, 1, $\frac{2}{3}$, $\frac{4}{9}$, &c.?
4. What is the sum of the infinite series, 1, $\frac{2}{5}$, $\frac{4}{25}$, $\frac{8}{125}$, &c.?

CHAPTER X.

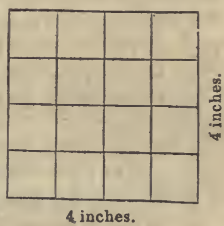
INVOLUTION AND EVOLUTION.

INVOLUTION.

ART. **229.** INVOLUTION, teaches the method of raising a number to any proposed power.

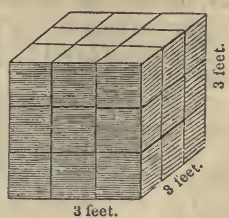
The number to be raised to a given power is called the *first power*, or ROOT. The product obtained by multiplying that number by itself, is called the SQUARE, or *second power* of that number.

REMARK.—We generally say, the *square* of a number, instead of the *second* power of that number, because the surface, or superficial contents of a geometrical square is obtained by multiplying the number of linear units expressing one of its sides by itself. Thus, the side of the adjacent figure is expressed by 4 linear units, (4 inches) and its superficial contents, by $4 \times 4 = 16$ *square inches*.



If the *square* or second power of a number be multiplied by the first power of that number, the product is called the CUBE, or third power of that number.

REMARK.—We generally say, the *cube* of a number, instead of the *third* power of that number, because the *solid contents* of a geometrical cube is expressed by the *third* power of the number expressing one of its sides. Thus, the solidity of the annexed cube is expressed by $3 \times 3 \times 3 = 27$ *solid feet*.



The power to which a number is to be involved is sometimes expressed by a small figure, called an EXPONENT or

index, placed above and a little to the right of that number. Thus,

$5^2 = 5 \times 5 = \mathbf{25}$ the square of **5**.
$5^3 = 5 \times 5 \times 5 = \mathbf{125}$ the cube of **5**.
$5^4 = 5 \times 5 \times 5 \times 5 = \mathbf{625}$ the fourth power of **5**.
&c., &c., &c.

The exponent of a quantity shows how many times that uantity *enters as a* FACTOR.

ART. **230.** A quantity is involved to any given power by *muliplying it by itself as many times as there are units in the exponent, less one.*

1. What is the square of each of the following numbers: 7, 8, 9, 12, 25, 38, 274, and 487?
2. What is the cube of 23, 84, 96, and 273 respectively?
3. What is the fourth power of 16, 18, 24, 147, and 263, respectively?
4. What is the fifth power of 3·2, 41·5, 82·5 and ·829, respectively?
5. What is the fifth power of $\frac{2}{3}$, $\frac{3}{5}$, $\frac{3}{4}$ and $\frac{1}{5}$, respectively?
6. What is the cube of $3\frac{1}{2}$, $2\frac{2}{3}$, $4\frac{1}{3}$, and $14\frac{2}{5}$, respectively?

EVOLUTION.

ART. **231.** EVOLUTION is the reverse of INVOLUTION. It teaches the method of resolving a number into *equal factors*, either of which is termed a ROOT.

The *square root* of **49** $(= 7 \times 7)$ is **7**, since 7 is one of the two equal factors of 49.

The *cube root* of **27** $(= 3 \times 3 \times 3)$ is **3**, since 3 is one of the three equal factors of 27.

Numbers whose roots can only be approximately obtained, are called *surd numbers.*

The *square root* is indicated by the symbol $\sqrt{}$, and is called the *radical sign.* Thus, $\sqrt{9}=3$; $\sqrt{\mathbf{25}}=5$.

The *cube root* is indicated by placing the figure ³ above the radical sign. Thus, $\sqrt[3]{\mathbf{27}}=3$; $\sqrt[3]{\mathbf{64}}=4$.

SQUARE ROOT.

ART. **232.** The *square root* of any number, which is not a surd, may be determined by

Resolving the number into its prime factors—the continued product of every other one of these different factors will be the root required.

1. What is the square root of 5184?

OPERATION.

$$5184 = 2 \times 2^* \times 2 \times 2^* \times 2 \times 2^* \times 3 \times 3^* \times 3 \times 3^*$$

EXPLANATION.—Every other one of the different prime factors of 5184 is marked by *; the product of which is $2 \times 2 \times 2 \times 3 \times 3 = 72$, the *square root* of 5184.

2. What is the square root of 900?
3. What is the square root of 18225?
4. What is the square root of 396900?

ART. **233.** The square root of any quantity which is not a surd, and is expressed by not more than *four* figures, can be ascertained by inspection.

First, square the nine digits respectively, and observe the terminating figure of each square number.

Digits.		*Their Squares.*
1^2	=	1
2^2	=	4
3^2	=	9
4^2	=	16
5^2	=	25
6^2	=	36
7^2	=	49
8^2	=	64
9^2	=	81

The terminating figures that are alike are linked together.

We observe that all square numbers end in 1, 4, 9, 6, or 5; also, if the number ends in 9, the figure in the root occupying the unit's place must be either 3 or 7; if in 4, the figure in the root occupying the unit's place must be either 2 or 8, &c. The figures occupying the *hundreds*, or the *hun-*

dred's and *thousand's* place, will enable us to determine the figure of the root occupying the ten's place; and by the excess of the given quantity above the square of the ten's figure, we are enabled to tell which of the two figures that will produce the terminating figure of the quantity, is the root.

1. What is the square root of 5184?

REMARK.—In accordance with what we have already learned, we know the figure in the root occupying unit's place must be 2 or 8; and the one occupying the ten's place must be 7, as its square, 49, is the largest square number, which is less than 51; and since the excess of the 51 above 49 is so small, we take the figure 2 for the unit's figure of the root. Hence, the square root of the above number is 72. Should the number have been 6084, then the excess of 60 above 49 would have been so great, we should have taken the 8 for the unit's figure of the root. Hence, we would have 78 for the square root of 6084.

2. What is the square root of 676?
3. What is the square root of 2209?
4. What is the square root of 1225?
5. What is the square root of 2916?
6. What is the square root of 3969?
7. What is the square root of 5041?
8. What is the square root of 7921?
9. What is the square root of 8464?
10. What is the square root of 9025?

ART. **234.** The square of 1, (the smallest digit,) is 1. The square of 9, (the largest digit,) is 81. Hence, the square of any digit is expressed by either *one* or *two* figures.

The square of 10 (the smallest number denoted by two figures,) is 100. The square of 99, (the largest number denoted by two figures,) is 9801. Hence, the square of any number denoted by two figures, is expressed by either *three* or *four* figures; in the same manner it may be shown, that the square of any number denoted by three figures, will be expressed by either *five* or *six* figures, &c.

Hence, the square of any number will contain twice as many figures as that number, or twice as many, less one. Therefore, *to extract the square root, we first separate the number into periods of two figures each, commencing at the right*

ART. **235.** As Evolution is the reverse of Involution, we will involve a few quantities by considering them decomposed into UNITS, TENS, HUNDREDS, &c., from which we will deduce a general rule for the extraction of the square root.

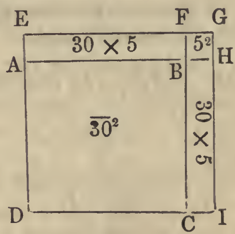
The square of a binomial, that is, a quantity consisting of two terms, is equal to, *The square of the first term, plus twice the first term into the second, plus the square of the second term.*

1. What is the square of 35 ?

$35 = 30 + 5$. Consider 30 the first term and 5 the second; then by the above rule we have, $\overline{35}^2 = (30+5)^2 = \overline{30}^2 + 2\times30\times5+5^2 = 1225$.

The evolution by multiplication is as follows :—

$$\begin{array}{l} 30 + 5 \\ 30 + 5 \\ \hline \overline{30}^2 + 30\times5 \\ \qquad\quad 30\times5 + 5^2 \\ \hline \overline{30}^2 + 2\times30\times5 + 5^2 \end{array}$$



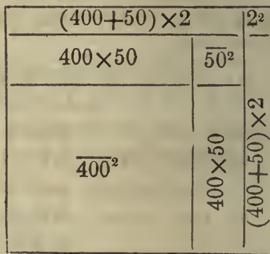
This involution may be geometrically illustrated thus :—Suppose the square ABCD, to be 30 inches each way; then its superficial contents is expressed by $\overline{30}^2$. This square may be increased by the two rectangles ABFE and BCIH, each equal in length to the side of the square, and in width to BH, (or 5,) the quantity by which the square has been increased; hence, the area of each of these rectangles is expressed by 30×5; also the little square BHGF, whose side is BH, (or 5); hence, its area is 5^2.

ART. **236.** The square of any polynominal is equal to, *The square of the first term, plus twice the first term into the second, plus the square of the second; plus twice the sum of the first two into the third, plus the square of the third; and so on.*

1. What is the square of 452?

$452 = 400 + 50 + 2$. Consider 400 the first term, 50 the second term, and 2 the third term; then by the above theorem we have, $(400+50+2)^2=\overline{400}^2+2\times 400 \times 50+\overline{50}^2+2\times(400+50)\times 2+2^2$.

The following diagram exhibits the above involution geometrically.



By reversing the above process of involution, we obtain for extracting the square root, the following

GENERAL RULE.

Commencing at the right, separate the number into periods of two numbers each.

Find the greatest square number in the first period on the left, and place its root at the right of the number, in the form of a quotient; also, on the left separating it from the number by a perpendicular line. Then subtract the square of this root from the period on the left; and to the remainder annex the second period; which will form the FIRST DIVIDEND.

Double the root already found, (which is placed at the left of the number); *to this product annex a cipher, and it will form the* FIRST TRIAL DIVISOR. *The number of times the trial divisor is contained in the first dividend, will be the next figure of the root, which must be added to the trial divisor, to form the* TRUE DIVISOR. *Multiply the true divisor by the figure of the root last obtained; subtract the product from the dividend, and to the remainder annex the next period for a* NEW DIVIDEND.

To the last divisor, add the last figure of the root found,

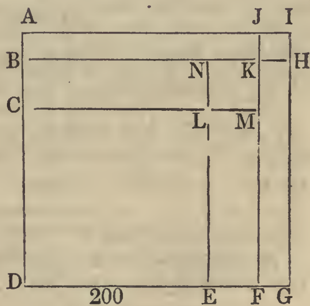
this sum with a cipher annexed will be the next TRIAL DIVISOR. *Then proceed as before, until all the periods have been brought down.*

NOTE.—When any dividend is not so large as its trial divisor, place a cipher for the next figure of the root; also, place a cipher at the right of the divisor, and form a new dividend by annexing a new period.

1. What must be the length of the side of a square pond that shall contain 54756 square feet?

OPERATION.

	NUMBER. Linear ft.	Sq. ft.	ROOT. Linear ft.
	200	$\dot{5}4\dot{7}5\dot{6}$	(200 + 30 + 4 = 234.
1st trial divisor,	400	40000	
true divisor,	430	14756	
2d trial divisor,	460	12900	
true divisor,	464	1856	
		1856	
		0	



EXPLANATION.—The requirement of the above question was to determine the side of a square that should contain 54756 square feet.

It is evident that the side of the square must be *more* than 200 linear feet, since the square of 200 is less than 54756; also, that it must be less than 300 linear feet, since the square of 300 is greater than 54756. Therefore, 2 is the greatest number whose square is contained in 5, (the left hand period,) and is the first, or *hundreds'* figure of the root.

Let CDEL be a square whose side is 200 linear feet. Then its area is $\overline{200}^2 = 40000$ square feet, which being taken from the given number, leaves 14756 square feet, to be added to the square DL. We first add the two rectangles CN and EM

each equal in length to the side of the square DL, which has already been found to be 200 feet. Therefore, the length of the two rectangles is 400 feet, which forms the 1st trial divisor. The 14756 being divided by the 1st trial divisor, gives a quotient of 30, which is the width of the rectangles CN and EM, also, the length of the side of the small square LMKN. Adding to 400, (the length of the two rectangles CN and EM,) 30, (the length or side of the square LK,) gives 430, the true divisor. Multiply 430, the length of these three pieces by 30, their width, gives 12900 square feet, the amount by which the square DL, has been increased. Subtract this amount from 14756 square feet leaves 1856 square feet, to be added to the square BDFK.

We now add to the square DK, the two rectangles BJ and FH, each equal in length to 200, (the side of the square DL,) plus 30, (the width of the rectangles just added to the square DL). Therefore, $2\ (200 + 30) = 460$ linear feet, is the length of the two rectangles BJ and FH, which forms the 2d trial divisor. Divide 1856, (the number of square feet remaining to be added to the square DK,) by the 2d trial divisor, gives 4 for the width of the two rectangles; also the side of the small square KHIJ. Hence, the length of the two rectangles BJ and FH, increased by the length of the square KHIJ, is 464, which forms the last true divisor; this length being multiplied by 4, the width of the three pieces, gives 1856 square feet, which being taken from 1856 leaves no remainder. Therefore, the square ADGI, the side of which is $200 + 30 + 4 = 234$ linear feet, contains 54756 square feet.

By omitting the ciphers the foregoing operation will take the following condensed form :—

OPERATION.

	NUMBER.	ROOT.
Linear ft.	Sq. ft.	Linear ft.
2	54756	(234
43	4	
464		
	147	
	129	
	1856	
	1856	
	0	

2. What is the square root of 85264?
3. What is the square root of 55696?
4. What is the square root of 1971216?
5. What is the square root of 5499025?
6. What is the square root of 269222464?
7. What is the square root of 6497004816?
8. What is the square root of 609596100?
9. What is the square root of 4164081009664?

REMARK.—The roots of the above numbers can also be determined by Art. 232.

ART. **237.** To extract the square root of a decimal.

Commencing at the decimal point, separate the number into periods of two figures each; then proceed according to the General Rule, and to the number annex ciphers, until the desired number of figures in the root are obtained.

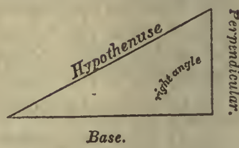
1. What is the square root of 22·3024?
2. What is the square root of 64·1601?
3. What is the square root of 84·65901?
4. What is the square root of 187·20924?
5. What is the square root of 5·296?
6. What is the square root of 2?
7. What is the square root of 3?
8. What is the square root of 5?
9. What is the square root of 6?
10. What is the square root of 7?
11. What is the square root of 9?

ATR. **238.** To extract the square root of a common fraction.

Reduce the fraction to its simplest form; then extract the root of the numerator and denominator separately, if they have an exact root; if not, reduce the fraction to a decimal, and proceed as in Art. 237.

1. What is the square root of $\frac{96}{294}$?
2. What is the square root of $\frac{2016}{2366}$?
3. What is the square root of $\frac{27}{48}$?
4. What is the square root of $\frac{3}{4}$?
5. What is the square root of $\frac{2}{3}$?

QUESTIONS INVOLVING THE PRINCIPLES OF SQUARE ROOT.



ART. **239.** A triangle is a figure having three sides, and therefore three angles. When one of the angles is right, like the corner of a square, (that is, contains 90°,) the triangle is called a RIGHT-ANGLED TRIANGLE. The side opposite the right-angle, is called the *hypothenuse;* one of the remaining two sides is called the *base*, and the other the *perpendicular.*

ART. **240.** Two sides of a right-angled triangle being given, the third side can be found by means of the following *theorem.*

It is an established theorem of geometry, that the square of the hypothenuse is equal to the SUM *of the* SQUARES *of the other two sides.*

Therefore, the square of one of the sides is equal to the square of the hypothenuse, diminished by the square of the other side.

1. How long must a ladder be to reach to the top of a tree 52 feet high when the foot of it is 39 feet from the tree?

OPERATION.

$$\overline{52}^2 = 2704$$
$$\overline{39}^2 = 1521$$

$\sqrt{4225} = 65$ feet, the length of the ladder.

2. It is ascertained that a ladder 95 feet in length, standing on the bank of a river 57 feet in width, reaches to the top of a tree standing on the opposite bank. What is the height of the tree?

3. Two ships start from the same place and sail, the one North and the other East. How far apart will they be in 6 days, providing they sail, at the rate of 72 and 96 miles an hour, respectively?

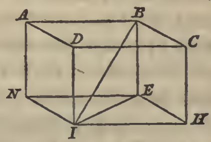
4. A man standing 39 paces, of 3 feet each, from a tree which is 95 feet high and 6 feet in diameter, shoots a pigeon from its top; how far did the ball move before it

reached the pigeon, providing the man's eye, the place from which the ball started, is five feet above the ground?

5. What is the distance between the opposite corners of a parallelopipedon, the length of which is 8 feet, and the width and depth, each 6 feet?

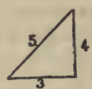
REMARK.—The question will be more readily comprehended by inspecting the following diagram

From the right-angled triangle IHE we determine the hypothenuse, IE. Then, from the right-angle triangle IEB, we determine the hypothenuse, IB, which is the distance between the opposite corners of the parallelopipedon, DE.



ART. **241.** The three smallest integers that can accurately express the length of the sides of a right-angled triangle, are 3, 4, and 5.

Thus—



If we multiply these three numbers by 2, it will give a right-angled triangle, the sides of which are 6, 8, and 10; if by 3, another, the sides of which are 9, 12, and 15. In the same way, any number of triangles may be obtained, the sides of which are expressed by integers.

Hence, by knowing two sides of a right-angled triangle, the sides of which are to each other as 3, 4, and 5 we can readily determine the remaining side, mentally.

1. What must be the length of a ladder to reach to the top of a tree, 48 feet high, when its foot is placed 36 feet from its base?

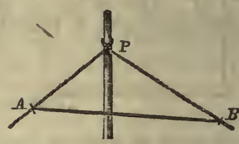
2. What is the distance, between the opposite corners of a rectangular field, the length of which is 32 rods, and the width 24 rods?

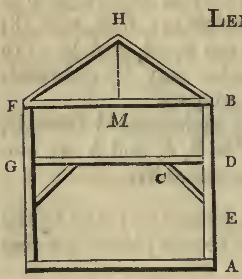
3. What is the length of a rectangular field, the distance between the opposite corners of which is 30 rods, and the width of which is 18 rods?

Mechanical Application of the Foregoing.

Art. **242.** Mechanics generally make use of a right-angled triangle, the sides of which are 6, 8, and 10 feet, respectively, in squaring the walls for the foundation of a building, &c. This is done by placing an upright stick where we design the corner of the building to be, with a cord about it so as to form a plain angle; then measure off 6 feet on one end of the cord, and 8 feet on the other, and holding the cord horizontal, place the terminating point of the 6 feet (which may be marked by sticking a pin through the cord,) at one extremity of a ten-foot pole, and the terminating point of the 8 feet at the other extremity. The triangle thus formed will be a right-angled triangle.

The following diagram will render the above remark more plain. P represents the upright stick, about which the cord is placed. PA, the 6 feet measured off, PB, the 8 feet, and AB, the ten-foot pole.





Length of Braces.

AB is the corner post of a building; DG a girth; and CE a brace. The triangle CDE is a right-angled triangle; hence, the length of the brace CE is found by extracting the square root of the *sum* of the squares of the two lengths DE and DC. (See Art. 240.)

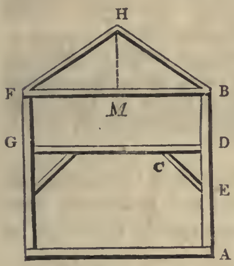
Art. **243.** *The length of any brace, when DC and DE are of the same length, is equal to the length DC + as many times* 5 *inches as DC is feet in length.* This fact is of great practical utility to carpenters.

1. What is the length of a brace, when the two sides DC and DE are each 3 feet long?

The length of the brace will be 3 feet + 3 times 5 inches, equal to 3 feet + 15 inches = 4 feet 3 inches.

2. If the sides DC and DE are each 4 feet long, $3\frac{1}{2}$ feet long, $4\frac{1}{2}$ feet long, 5 feet long, $5\frac{1}{2}$ feet long, or 6 feet long, what would be the length of the braces for the several conditions, respectively.

Length of Rafters, &c.



ART. **244.** To find the length of a rafter.

FB is called the base line of the roof, and HM the height of the pitch of the roof.

If the height of the pitch is equal to *one-half* of the base line; HM= MB; hence, the length of the rafter, HB, is found in the same manner we found the length of a brace, under Art. 243.

A roof is said to be *one-fourth*, *two-fifths*, *three-sevenths*, &c., *pitch*, when HM = $\frac{1}{4}$, $\frac{2}{5}$, $\frac{3}{7}$, &c., of the base line, FB. The length of the rafters of such roofs, is found *by extracting the square root of the* SUM *of the squares of HM and MB.* (See Art. 240.)

1. In a three-eighths pitch roof, the base of which is 40 feet, what is the length of the rafters?

SOLUTION.—In this example the height of the pitch HM= 15 feet.

The MB = 20 feet.

$\overline{15}^2 = 225$ feet.

$\overline{20}^2 = 400$ feet.

The square root of $\overline{625} = 25$, the length of the rafters.

2. In a two-seventh pitch roof, the length of the base of which is 28 feet, what is the length of the rafters?

3. In a one-fourth pitch roof, the base of which is 24 feet, what is the length of the rafters?

4. In a three-fifths pitch roof, the base of which is 40 feet, what is the length of the rafters?

ART. **245.** *It is an established theorem of geometry, that all similar surfaces, or areas, are to each other as the squares of their like dimensions.*

Hence, *the like dimensions of similar figures are to each other as the square roots of their areas.*

1. There are two circular fish-ponds; one of which is 20 rods in diameter, and the other 4 rods in diameter. How much more surface in the one than in the other?

2. A farmer has a rectangular piece of land containing $6\frac{1}{2}$ acres, the width of which is 10 rods, and the length 100 rods. His neighbor has a similar piece of land containing 9 acres. Required the length and breadth of his neighbor's piece of land.

3. Suppose a horse to be tied to a post in the centre of a field, by a rope 7·13 rods in length, and is thereby enabled to graze upon 1 acre. How long should the rope be to allow it to graze upon $6\frac{1}{2}$ acres?

4. By observation I find that $11\frac{1}{9}$ gallons of water will flow through an orifice of $1\frac{1}{9}$ inches in diameter in 1 second. How large should the orifice be so as to discharge $2\frac{1}{4}$ gallons in the same time.

5. If it require $156\frac{1}{4}$ yards of carpet to cover a floor that is 25 feet in length, and $20\frac{5}{6}$ feet in width; what must be the dimensions of a similarly shaped floor, that requires $56\frac{1}{4}$ yards of the same kind of carpet to cover?

6. Five men purchased a grindstone 40 inches in diameter. How much of the diameter must each grind off, so as to have $\frac{1}{5}$ of the stone?

REMARK.—After the first has ground off his share, $\frac{4}{5}$ of the stone remains, and its diameter will be $40\sqrt{\frac{4}{5}} = 8\sqrt{20}$, &c.

ART. **246.** When the base and the sum of the height and hypothenuse of a right-angled triangle are given, to find the hypothenuse:

Add the square of the height and hypothenuse to the square of the base, and divide their sum by twice the height and hypothenuse.

1. There is a tree 80 feet in height, standing by the bank of a river 50 feet in width. Where must this tree break off, so that the top will reach across the river, while the broken parts remain in contact?

CUBE ROOT.

ART. **247.** Whenever the cube root of a quantity is expressed by a whole number, it may be found by,

Resolving the number into its prime factors. The product of every third factor of these different factors, will be the root required.

1. What is the cube root of 729000?

OPERATION.

```
 2)729000
   ------
 2)364500
   ------
*2)182250
   ------
  3)91125
   ------
  3)30375
   ------
 *3)10125
   ------
   3)3375
   ------
   3)1125
   ------
   *3)375
   ------
    5)125
    -----
     5)25
     ----
       *5
```

EXPLANATION.—Taking the continued product of every third one of these different factors, (which are marked by *,) we have $2 \times 3 \times 3 \times 5 = 90$, which is the cube root of 729000.

2. What is the cube root of 4741632?
3. What is the cube root of 98611128?
4. What is the cube root of 621875?

5. What is the cube root of 23887872 ?
6. What is the cube root of 5639752 ?
7. What is the cube root of 5936493568 ?

ART. **248.** The cube root of any quantity which is not a surd and is expressed by not more than *six* figures, can be ascertained by inspection.

First, cube the nine digits respectively, and observe the terminating figure of each cube number.

Digits.	*Their Cubes.*
$1^3 =$	1
$2^3 =$	8
$3^3 =$	2 7
$4^3 =$	6 4
$5^3 =$	12 5
$6^3 =$	21 6
$7^3 =$	34 3
$8^3 =$	51 2
$9^3 =$	72 9

It will be observed that the terminating figure of each of the cubes of the nine digits is either 1, 2, 3, 4, 5, 6, 7, 8, or 9; hence, every cube number must terminate with one of the nine digits, consequently the figure of the root occupying the unit's place is readily determined by inspection. The figure of the root occupying the ten's place is determined by inspecting the number, considered as units, that preceed the first three figures.

1. What is the cube root of 614125 ?

EXPLANATION.—As this number ends in 5 the figure in the root occupying the unit's place must be 5. 8 is the largest number, the cube of which is less than the number expressed by the figures on the left of the first three figures, which is 614; hence, the cube root of 614125 is 85.

2. What is the cube root of 35937 ?
3. What is the cube root of 226981 ?
4. What is the cube root of 117649 ?
5. What is the cube root of 50653 ?
6. What is the cube root of 110592 ?
7. What is the cube root of 405224 ?
8. What is the cube root of 438976 ?
9. What is the cube root of 778688 ?

ART. **249.** Before we attempt to explain the usual method of extracting the CUBE ROOT, we will involve a number, consisting of *units* and *tens*, and of *units*, *tens*, and *hundreds* to its third power.

REMARK.—The cube of 1, the smallest digit is 1. The cube of 9, the largest digit is 729. Therefore, the cube of any digit is expressed by *one, two*, or *three* figures.

The cube of 10, the smallest number denoted by two figures, is 1000. The cube of 99, the largest number denoted by *two* figures, is 970299. Therefore, the cube of any number denoted by two figures is expressed by FOUR, FIVE, or SIX figures. In the same manner it may be shown, that the cube of a number denoted by *three* figures is expressed by SEVEN, EIGHT, or NINE, figures, &c.

Hence, if a number be denoted by *one, two*, or *three* figures, its cube root will be expressed by *one* figure; if by *four, five*, or *six* figures, its cube root will be expressed by *two* figures, &c.

In general the cube will contain THREE *times as many figures as the root, or* THREE *times as many less* ONE *or* TWO. Therefore, to extract the cube root, we first *separate the number into periods of three figures each, commencing at the right.*

ART. **250.** The cube of a Binomial, (that is, a number consisting of two terms,) is, *the cube of the first, or* left hand term, *plus three times the square of the first term into the second, plus three times the first term into the square of the second term, plus the cube of the second term.*

In general, the cube of any Polynomial is equal to the *cube of the first*, or left-hand term, *plus three times the square of the first term into the second, plus three times the first into the square of the second, plus the cube of the second; plus three times the square of the* SUM *of the* FIRST TWO *into the third, plus three times the* SUM *of the first two into the square of the third, plus the cube of the third, &c.*

1. What is the cube of 89?

$89=80+9$; and by Art. 250, we have

$$(80+9)^3=\overline{80}^3+3\times\overline{80}^2\times 9+3\times 80\times 9^2+9^3$$

2. What is the cube of 357?

$357=300+50+7$; therefore;

$$(300+50+7)^3=\overline{300}^3+3\times\overline{300}^2\times 50+3\times 300\times \overline{50}^2+\overline{50}^3\times 3(300+50)^2\times 7+3(300+50)\times 7^2+7^3$$

3. What is the cube of 468?

$468=400+60+8$; $\therefore$,

$$(400+60+8)^3=\overline{400}^3+3\times\overline{400}^2\times 60+3\times 400\times\overline{60}^2+\overline{60}^3+3(400+60)^2\times 8+3(400+60)\times 8^2+8^3$$

The involution of example 1, by multiplication, is as follows:—(the second, &c., is similar to it).

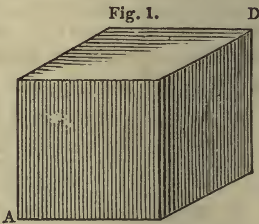
$$80+9$$
$$80+9$$
$$\overline{80}^2+80\times9$$
$$80\times9+9^2$$
$$(80+9)^2=\overline{80}^2+2\times80\times9+9^2$$
$$80+9$$
$$\overline{80}^3+2\times\overline{80}^2\times9+80\times9^2$$
$$\overline{80}^2\times9+2\times80\times9^2+9^3$$
$$(80+9)^3=\overline{80}^3+3\times\overline{80}^2\times9+3\times80\times9^2+9^3$$

We will now illustrate *geometrically* the involution of the first example.

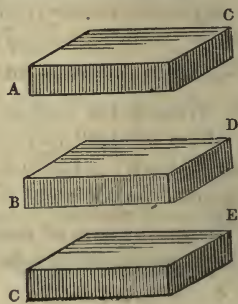
How many cubic feet in a cube, each side of which is 89 feet?

$89=80+9$.

Fig. 1.

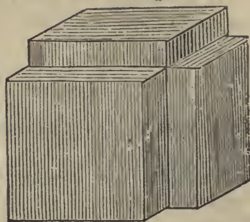


Suppose each side of the cube AD, (fig. 1,) to be 80 feet; then its solid contents will be $\overline{80}^3=512000$ cubic feet.

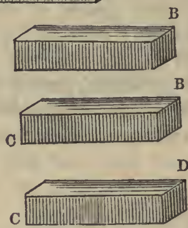


To increase the size of the cube AD, we will first add the three square slabs, AC, BD, and CE, each of the sides of which is 80 feet, (the side of the cube AD,) and the thickness of each, 9 feet. Hence, the solid contents of one of these square slabs is $\overline{80}^2\times9$, and the three, $3\times\overline{80}^2\times9=172800$ cubic feet.

Fig. 2.



The cube AD, increased by the three square slabs, AC, BD, and CE, is represented by Fig. 2;—the contents of which is 512000 $+ 172800 = 684800$ cubic feet.



We will now increase Fig. 2, by the three *equal* corner-pieces, AB, BC, and CD, the length of each being 80 feet, (the side of the cube AD,) and the *width* and *thickness* of each, 9 feet. Hence, the solid contents of one of these pieces is 80×9^2, and of the three, $3 \times 80 \times 9^2 = 19440$ cubic feet.

Fig. 3.

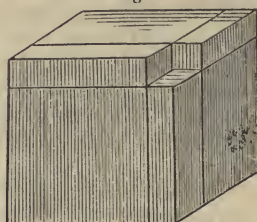
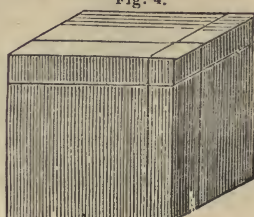


Fig. 2, increased by the three pieces, AB, BC, and CD, is represented by Fig. 3;—the contents of which is $512000 + 172800 \times 19440 = 704240$ cubic feet.

We will now increase Fig. 3, by the small cube XY, each side of which is 9 feet; therefore, its contents is $9^3 = 729$ cubic feet;

Fig. 4.



This cube is then represented by Figure 4, the contents of which is $512000 + 172800 + 19440 + 729 = 704969$ cubic feet, and the side of which is 89 feet.

By reversing the above process, we obtain for extracting the cube root, the following

GENERAL RULE.

Commencing at units, separate the number into periods of three figures each.

Then find the largest digit, the cube of which shall not exceed the left-hand period. Place this digit, which is called the first figure of the root, on the right, in the form of a quotient; also, on the left, for the first term of a first column, and its square for the first term of a second column, and from the left-hand period of the given number, subtract its cube. Then to the remainder, annex the next period, for the FIRST DIVIDEND. *Now double the term in the first column, for its second term, and add its product into the root already found, to the first term of the second column, for the first* TRIAL DIVISOR. *Consider two ciphers annexed to the trial divisor, and write the number of times it is contained in the dividend, for the next figure of the root; also, annex it to the sum of the last term in the first column, and the first figure of the root;—this will be the next term of the first column. Add the product of this term into the digit of the root last found, advancing it two places to the right, to the last term of the second column, for its next term; this will be the* TRUE DIVISOR. *From the* DIVIDEND, *subtract the product of the true divisor into the digit of the root found; and to the remainder annex the next period, for the second dividend.*

Proceed in a similar way until all the periods have been used.

REMARK.—By carefully examining the foregoing involution, the pupil will be able to deduce other rules for the extraction of the cube root, some of which may perhaps, appear more plain than the one I have just given, as this is more readily deduced from Algebraic involutions. I have given this rule, as it will be less laborious to extract the cube root of large numbers by it, than by many other rules usually given; also, because it keeps distinct the three geometrical magnitudes—*lines*, *surfaces*, and *solids*.

The *first rule*, however, is the most simple, and will be found of much importance in reducing *surd quantities* to their simplest form, (as will hereafter be explained,) or in determining the roots of *rational* quantities.

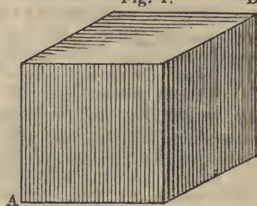
1. What is the cube root of 704969?

OPERATION.

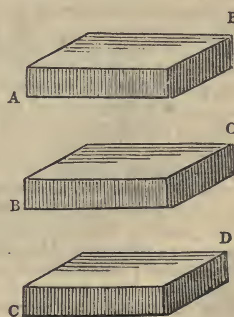
FIRST COL. *Linear feet.*	SECOND COL. *Square feet.*	NUMBER. *Cubic feet.*	ROOT. *Linear feet.*
80	6400	$70\dot{4}96\dot{9}$	(80 + 9 = 89.
160	19200, trial divisor.	512000	
249	21441, true divisor.	192969,	1st dividend.
		192969	
		0	

EXPLANATION.—We first find the greatest cube contained in the left-hand period. We know that this number must be more than 80, since $\overline{80}^3 = 512000$, which is less than 704969; also, that it must be less than 900, since $\overline{90}^3 = 729000$, which is greater than 704969. Hence, the first, or left-hand figures of the root, is 8; whose cube is 512, which is the greatest cube contained in 704, the first or left-hand period.

Fig. 1.



Let each side of the cube AD, represented by Fig. 1, be 80 linear feet; then its cubical contents will be $\overline{80}^2 = 512000$ cubic ft., and 704969—512000=192969 cubic feet, which is still to be added to the cube AD.



We first add the three square slab pieces AB, BC, and CD, whose length and breadth are each respectively 80 feet, (the side of the cube AD.) The area of the face of the first piece, AB, is $\overline{80}^2 = 6400$ square feet. The length of the other two pieces, BC, and CD, is $80 + 80 = 160$ feet, and their width 80 feet. Hence, their superficial contents is $160 \times 80 = 12800$ square feet, which added to 6400 square feet, the superficial contents of the piece, AB, gives 19200 square

feet, the superficial contents of the three pieces, AB, BC, and CD. As these three square slabs make up by far the greatest amount of the whole increase, if we divide 192969, (the number of cubic feet remaining to be added,) by 192000, the number of square feet in the three pieces, AB, BC, and CD, (which may be called the trial divisor,) it will give their thickness; which we find to be 9 feet.

Fig. 2.

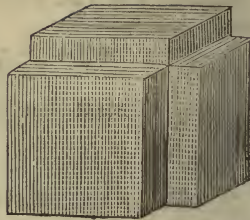
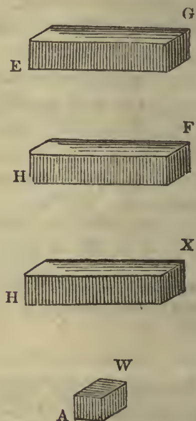
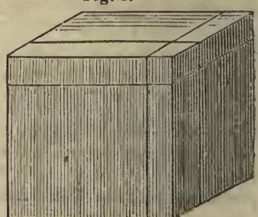


Figure 2, represents the cube AD, with the three pieces AB, BC, and CD, added.



We now add the three corner-pieces, EG, HF, and HX, whose lengths are respectively 80 feet, (the side of the cube AD,) and whose width and thickness are each 9 feet respectively; also, the corner-piece AW, whose length, width, and thickness, are each 9 feet. Therefore, the length of the three pieces, EG, HF, and HX, is 240 feet; which being increased by the length of the corner-piece, AW, gives 249 feet for the length of the four pieces. Their width is 9 feet; therefore, $249 \times 9 = 2241$ square feet, is their superficial contents; which being increased by 19200 square feet, the superficial contents of the three pieces already added on, gives 21441 square feet, (the superficial contents of the seven pieces added on,) which being multiplied by 9, their thickness, gives 192969 cubic feet, their solid contents, which being subtracted from 192969 cubic feet, the quantity that remained to be added to the cube AD, leaves no remainder; therefore, a cube represented by Figure 3, whose side is $80 + 9 = 89$ feet, will cantain 704969 cubic feet.

Fig. 3.



This work may be condensed by omitting the ciphers and unimportant terms.

First Col.	*Second Col.*	*Number. Root.*
8	64	704969(89
16	192	512
249	21441	192969
		192969

2. What is the cube root of 12895213625 ?

REMARK.—To render the method of extracting the cube root familiar, when there are a number of figures in the root, we will perform the above example

First Col.	*Second Col.*	*Number.*	*Root.*
2	4	12895213625(	2345
		8	
4	12		
63	1389	4895	
66	1587	4167	
694	161476	728213	
698	164268	645904	
7025	16461925	82309625	
		82309625	

REMARK.—When the trial divisor is greater than its corresponding dividend, place 0 for the next figure of the root, and bring down the next period. Then use the same trial divisor with two more ciphers annexed.

3. What is the cube root of 469640998917 ?

REMARK.—Whenever the number has not an exact root, there will be a remainder after the last period has been brought down. The process may be continued, and the true root more nearly obtained, by annexing ciphers to new periods. The figures thus obtained will be decimals.

4. What is the cube root of 1860867 ?
5. What is the cube root of 469640998917 ?
6. What is the cube root of 58050510848 ?
7. What is the cube root of 84672374 ?
8. What is the cube root of 3 ?
9. What is the cube root of 5 ?
10. What is the cube root of 7 ?
11. What is the cube root of 4937801347510680732948 ?

ART. **251.** To extract the cube root of a decimal. First, *commencing at the decimal point, separate the numbers*

into periods of three figures each; if necessary, annex ciphers, so that the decimal may be separated into equal periods. Then proceed as usual.

12. What is the cube root of 561·515625?
13. What is the cube root of 460·099648?
14. What is the cube root of ·117649?
15. What is the cube root of 7·256313856?
16. What is the cube root of 47·86?

Art. **252.** To extract the cube root of a fraction, or mixed number, first reduce either of them to its simplest form; then find the root of the numerator and denominator separately, if their roots can be accurately found; if not, reduce the fraction to a decimal, and extract the root as above directed.

17. What is the cube root of $\frac{2197}{5832}$?
18. What is the cube root of $\frac{15252392}{51064811}$?
19. What is the cube root of $\frac{2197}{64}$?

Art. **253.** *A rational quantity* is one that can be expressed in numbers. Thus, $\sqrt[3]{27}$ is *rational.*

An *irrational* or *surd* quantity is one that has not an exact root, or which cannot be expressed in numbers. Thus, $\sqrt[3]{3}$ is a *surd.*

When the cube root of a large quantity is required, it will be found more convenient to find the root of the *rational* part of the number by Art. 247, and the root of the *surd* part by General Rule.

Thus, $\sqrt[3]{40}=\sqrt[3]{2\times2\times2\times5}=2\sqrt[3]{5}=2\times1{\cdot}709976+=$ $3{\cdot}419952+$.

1. What is the cube root of 19208000?

OPERATION.

$$\begin{array}{r} 2)\underline{19208000} \\ 2)\underline{9604000} \\ *2)\underline{4802000} \end{array}$$

[over.

```
  2)2401000
  2)1200500
 *2)600250
  5)300125
  5)60025
 *5)12005
  7)2401
  7)343
 *7)49
    7
```

REMARK.—Every third factor is marked thus, *. Their product will be the cube root of the rational part, which is $2\times2\times5\times7=140$. This multiplied by $\sqrt[3]{7}$, will equal the cube root of the above number. Thus, $140\sqrt[3]{7}=140\times1{\cdot}912931+$ $=267{\cdot}81034+$.

2. What is the cube root of 128625?
3. What is the cube root of 61631955000?
4. What is the cube root of 257250?
5. What is the cube root of 84035?
6. What is the cube root of 2401000?

PRACTICAL QUESTIONS IN CUBE ROOT.

ART. **253.** *It is a theorem of geometry, that all similar solids are to each other as the* CUBES *of their like dimensions. Therefore,*

The dimensions of similar solids are to each other as the CUBE ROOTS *of their solidity.*

1. If a ball of iron 2 inches in diameter weigh 5 pounds, what will a ball of the same metal weigh, the diameter of which is 7 inches.

OPERATION.

$2^3 : 7^3 :: 5 :$ weight required.
$8 : 343 :: 5 : 214\frac{3}{8}$ pounds.

2. If the Earth is 8000 miles in diameter, and Mercury 3200 miles, how many times larger is the Earth than Mercury?

3. If a ball $\frac{2}{3}$ of an inch in diameter weigh $\frac{1}{9}$ of a pound, what must be the diameter of another ball of the same metal to weigh 24 pounds?

4. What is the diameter of a globe of gold that is worth \$9785036·80, providing a pound of gold (avoirdupois weight,) is worth \$256?

It will be remembered that the specific quantity of gold is $18\frac{1}{4}$; and that a cubic foot of water weighs $62\frac{1}{2}$ pounds.

5. If a man $5\frac{1}{2}$ feet in height, weigh 125 pounds, how tall is that man who weighs 216 pounds?

6. A cask 60 inches long and 36 inches at the bung diameter, contains a certain number of gallons; what must be the dimensions of a similar cask that shall contain $\frac{1}{8}$ as much?

7. A carpenter wishes to make a cubical cistern that shall contain 74088 cubic feet of water; what must be the length of one of its sides?

8. If a cellar 12 feet long, 8 feet deep and 8 feet wide contain 768 cubic feet; what is the dimensions of a similar cellar that shall contain 20736 cubic feet?

9. What will be the dimensions of a rectangular box, which shall contain 1845480 cubic feet, the length, breadth, and depth being to each other as 7, 5, and 3?

10. A ball of fine thread 5 inches in diameter is owned by five women; what portion of the diameter must each wind off so as to share equally of the thread?

GAUGING.

ART. **254.** *Gauging* teaches the method of finding the contents of any regular vessel, in gallons, bushels, &c.

ART. **255.** To find the number of gallons or bushels in a square vessel.

Take the dimensions in inches, and divide the PRODUCT *arising from multiplying the* LENGTH, BREADTH, *and* HEIGHT *together by* 282 *for ale gallons,* 231 *for wine gallons, and* 2150·42 *for bushels.*

1. How many wine gallons will a cubical box contain, that is 5 feet long, $2\frac{1}{2}$ feet wide and 2 feet high?

2. How many ale gallons will a vess el contain, that is 9 feet long, 4 feet wide, and 3 feet high?

3. How many bushel of grain will a bin contain, that is 16 feet long, 12 feet wide, and 6 feet high?

ART. **256.** To find the contents of casks.

Multiply the product of the square of the mean diameter and the length in inches, by ·0034, *and it will give the contents in wine gallons;* if by ·0028 instead of ·0034, it will give the contents in beer gallons.

REMARK.—The mean diameter is equal to the head diameter incraased by $\frac{7}{10}$ of the difference between the head and bung diameters when the staves are *much* curved, or by adding $\frac{1}{2}$ the difference, when but *little* curved; and by adding $\frac{13}{20}$ when they are of a mediam curve.

1. How many wine gallons does a cask contain whose length is 34 inches, bung diameter 28 inches, and head diameter 20 inches?

2. How many wine gallons does a cask contain, whose length is 45 inches, bung diameter 32 inches and its head diameter 22 inches!

ART. **257.** To find the contents of a round vessel, wider at one end than the other.

To $\frac{1}{3}$ of the square of the difference of the diameters, add their product and multiply this sum by the height. Then multiply by ·0034 *for wine gallons, and by* ·0028 *for ale or beer.*

1. How many wine gallons will a vessel contain, that is 48 inches in diameter at the bottom and 32 inches at the top; the length of which is 60 inches.

2. How many beer gallons will a tub contain, that is 40 inches in diameter at the bottom and 30 inches at the top; the length of which is 48 inches.

CHAPTER XI.

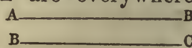
MENSURATION.

GEOMETRICAL DEFINATIONS.

1. A POINT is that which has position, but not magnitude.

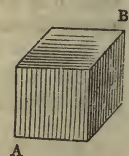
2. A LINE is length without width or thickness, as AB.

3. PARALLEL LINES are those which are everywhere equally distant, as AB and BC.



4. A SURFACE is that which has length and breadth without thickness, as ABCD.

5. A SOLID BODY is that which has length, breadth, and thickness; and therefore combines the three dimensions of extension, as AB.

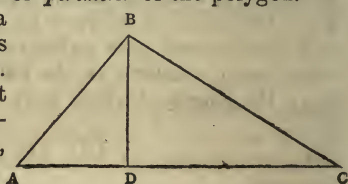


PLANE FIGURES.

ART. **258.** 1. A PLANE FIGURE is a plane surface terminated on all sides by lines, either straight or curved.

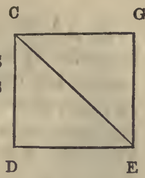
2. A POLYGON, or *rectilineal figure*, is a plane terminated on all sides by straight lines. The sum of these bounding lines is called the *contour* or *perimeter* of the polygon.

3. A TRIANGLE is a figure having three sides and three angles, as ABC. Its *altitude* is a line let fall from the vertex perpendicular to the base, as BD.

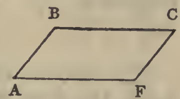


4. A SQUARE is a figure that has all of its sides equal, and its angles right-angles, as CDEG.

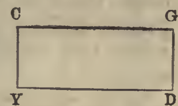
The line CE is called its *diagonal.*



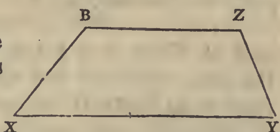
5. A PARALLELOGRAM is a figure that has its opposite sides parallel, as ABCF.



6. A RECTANGLE is an equiangular parallelogram, as CYDG.



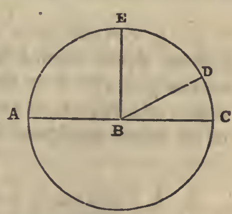
7. A TRAPEZOID is a figure that has only two of its sides parallel, as XYBZ.



REMARK.—It has already been remarked, that any figure, the sides of which are terminated by straight lines, is called a POLYGON.

A polygon of *three* sides is called a TRIANGLE; that of *four* sides a QUADRILATERAL; that of *five* sides, a PENTAGON; that of *six*, a HEXAGON; that of *seven*, a HEPTAGON; that of *eight*, an OCTAGON; that of *nine*, a NONAGON; that of *ten*, a DECAGON; that of *twelve*, a DODECAGON, &c.

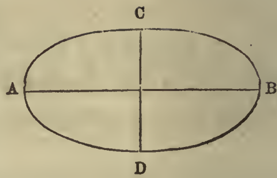
A CIRCLE is a plane, terminated by a curved line, every point of which is equally distant from a point within, called the centre. The curved line is called the *circumference.*



The DIAMETER of a circle is a line passing through the centre, and terminated by the circumference, as AC.

The RADIUS of a circle, is a line drawn from its centre to the circumference, as BD, BE, &c.; hence, it is half the diameter.

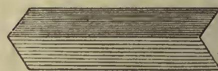
9. An ELLIPSE is a plane bounded by a curved line, the sum of the distances from every point of which to two given points, is equal to a given line. The line AB, is called the *transverse*, and CD, the *conjugate* axis.



SOLID FIGURES.

ART. **259.** A PRISM is a solid, the sides of which are parallelograms, and the ends equal and parallel polygons, as figure A.

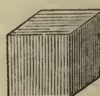
A



REMARKS.—When the ends of a Prism are triangular, it is called a *triangular prism;* when they are squares, it is called a *square prism*, &c.

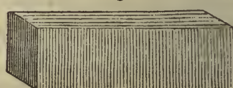
2. A CUBE is a *prism*, all the sides of which are *equal squares*, as figure B.

B



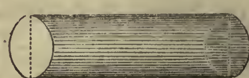
3. A PARALLELOPIPEDON is a prism, the ends of which are parallelograms, as figure C.

C



4. A CYLINDER is a solid, having equal circles for ends, and is generated by the revolution of a rectangle about one of its sides; as figure D

D

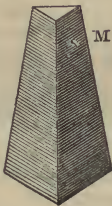


5. A PYRAMID is a solid, having for its base a plane rectilinear figure; and for its sides triangles, whose vertices meet in a point at the top, called the *vertex* of the pyramid. Figure E represents a triangular pyramid.

E

F

6. A CONE is a solid, having for its base a circle, and tapers uniformly to a point at the top. Figure F represents a cone.



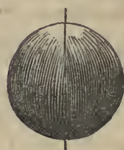
M

N

7. A FRUSTUM of a *pyramid*, or a *cone*, is the part that remains after cutting off the top by a plane parallel of the base.

Fig. M represents the frustum of a pyramid; and N, that of a cone.

X



8. A SPHERE is a solid, bounded by a convex surface, every point of which being equally distant from a point within called a center. Figure X represents a sphere.

MENSURATION OF SURFACES, &c.

ART. **260.** The pupil is referred to Geometry for the demonstration of the following rules for measuring surfaces, solids, &c.

ART. **261.** The *area* of a figure is the number of square inches, feet, or yards, &c., which it contains.

PROBLEM 1.—Given the base and altitude of a triangle, to find its area.

Multiply the base by half the altitude.

1. What is the area of a triangle whose base is 9 feet, and altitude 4 feet?

2. What is the area of a triangle whose base is 36 rods, and altitude 16 rods?

PROBLEM 2.—Given the three sides of a triangle, to find its area.

From half of the sum of the three sides, subtract each side separately; and the square root of the continued product of these three remainders and the half-sum will be the area.

1. What is the area of a triangle whose sides are, respectively, 12, 18, and 20 feet?

2. What is the area of a triangular field whose sides are respectively 40, 30, and 50 rods?

PROBLEM 3.—Given the sides of a rectangle, or square, to find its area.

Multiply the length by the width.

1. How many square feet of boards will be required to floor a room that is 36 feet long and 18 feet wide?

2. How many acres in a rectangular piece of land 460 rods long and 380 wide?

3, How many more acres in a piece of land 160 rods square, than in a rectangular piece 40 rods in length and 32 rods in width.

PROBLEM 4.—Given the base and altitude of a parallelogram, to find its area.

Multiply the base by the altitude.

1. What is the area of a parallelogram whose length is 28 feet, and altitude 22 feet?

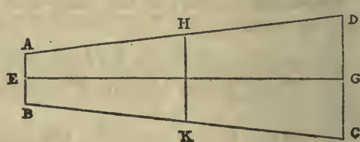
PROBLEM 5.—Given the altitude and the parallel bases of a trapezoid, to find its area.

Multiply the altitude by half the sum of its parallel sides.

1. What is the area of a trapezoidal field, the parallel sides of which are 18 and 24 rods respectively;—the perpendicular distance between these sides being 8 rods?

REMARK.—This rule is of practical use to lumbermen in measuring boards.

Let ABCD be a tapering board, and EG its length. The half sum of its parallel sides, AB and CD, is HK, the width of the board at the middle point. Its area, therefore, is expressed by HK×EG.



2. How many square feet in a tapering board 36 feet long, 18 inches wide at one end and 32 in. at the other?

PROBLEM 6.—Given the diameter of a circle to find its circumference.

Multiply the diameter by 3·1416.

1. What is the circumference of a circle whose diameter is 15 feet?

2. What is the circumference of a circle whose diameter is 24 rods?

REMARK.—The true ratio of the circumference of a circle to its diameter has never yet been found. Its approximate value has been extended to more than 200 places. A man by the name of *Van Ceulen* first extended the approximation to 36 places by means of continually bisecting the arc of a circle. This was considered so great an achievement that the 36 numbers expressing the circumference of a circle whose diameter is 1, was engraved on his tomb-stone.

The following are the numbers:—

3·14159265358979323846264338327950288 4

PROBLEM 7.—Given the circumference of a circle to find its diameter.

Divide the circumference by 3·1416.

1. What is the diameter of a circle whose circumference is 62·832 feet?

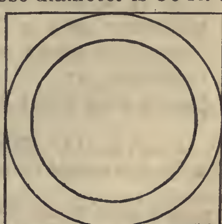
2. What is the diameter of a circle whose circumference is 48·5 feet?

PROBLEM 8.—Given the diameter of a circle, to find its area.

Multiply the square of the diameter by ·7854.

1. What is the area of a circle whose diameter is 15 inches?

2. What is the area of a circle whose diameter is 36 ft.?



REMARK.—From the above rule, we can readily determine the area contained between two concentric circumferences.

It is also worthy of remark that the area of a square is to the area of an inscribed circle, as 1 is to 0.7854.

PROBLEM 9.—Given the diameter of a circle, to find the side f an equal square.

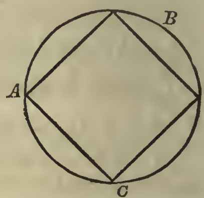
Multiply the diameter by .8862.

1. A gentleman has a circular fish-pond that is 8 rods in diameter ; what must be the side of a square pond that shall contain the same area ?

2. I have a circular piece of board that is 36 inches in diameter ; what must be the side of a square board that shall contain the same area ?

PROBLEM 10.—Given the diameter of a circle to find the side of an inscribed square.

Multiply the diameter by 0.7071.



1. Required the side of a square that can be inscribed in the circle ABC, whose diameter is 24 feet.

2. How large a square stick can be sawn from a piece of round timber, that is 42 inches in diameter ?

PROBLEM 11.—Given the diameters of an ellipse, to find its area.

Multiply the product of its two axes by 0·7854.

1. How many square rods in an elliptical garden, whose transverse axis is 280 feet, and conjugate axis 210 feet ?

2. How many square feet in an elliptical table, the transverse axis of which is 6 feet 9 inches, and the conjugate axis 3 feet 6 inches ?

PROBLEM 12.—Given the length and the circumference of a cylinder, to find its surface.

Multiply its circumference by its length, and to the product add the area of the two bases.

1. What is the surface of a cylinder whose length is 22 feet, and diameter $4\frac{1}{2}$ feet?

PROBLEM 13.—Given the length and the perimeter of the base of a prism, or parallelopipedon, to find its surface.

Multiply its length by the perimeter of its base, and to the product add the areas of the two ends.

1. What is the surface of a square prism whose side is 2 foot 8 inches, and length 16 feet?

2. What is the surface of a triangular prism whose length is 12 feet, and whose sides are, respectively, 3, 4, and 5 feet?

PROBLEM 14.—Given the side of a cube to find the area of its surface.

Multiply the area of one of its sides by 6.

1. What is the area of a cubic block, the side of which is 12 feet?

PROBLEM 15.—Given the slant height and the sides of the base of a pyramid to find its area.

To the area of the triangles that form its sides, add the area of the base.

1. What is the area of a triangular pyramid, the slant height of which is 12 feet, and the sides of its base 3, 4 and 5 feet, respectively?

2. What is the area of a square pyramid, the slant height of which is 35 feet, and the sides of its base 10 feet?

PROBLEM 16.—Given the slant height and the diameter of a cone, to find its surface.

Multiply the half sum of the slant height and the radius of the base by the circumference of the base.

1. What is the surface of a cone, the slant height of which is 47 feet and the diameter 18 feet?

PROBLEM 17.—Given the perimeter of the bases and the

slant height of the frustum of a pyramid, or cone, to find the area.

Multiply the half sum of the perimeter of the bases by the slant height, and to the product add the sum of the areas of the two bases.

1. Suppose the slant height of a square frustum is 24 feet, the side of the base 8 feet, and of the upper base or top 4 feet; what is its surface?
2. Suppose the slant height of the frustum of a cone to be 40 feet, the diameter of the base 15 feet, and that of the top 5 feet; what is its whole surface?

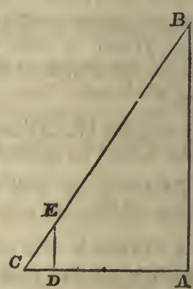
PROBLEM 18.—Given the diameter of a sphere to find its area or convex surface.

Multiply its circumference by its diameter. Or, which is the same thing,

Multiply the square of the diameter by 3·1416.

1. What is the surface of a sphere 48 inches in diameter?
2. What is the area of the earth's surface, supposing it to be 8000 miles in diameter?

ART. **262.** We can readily determine the distance to any visible object by means of a right-angled triangle.

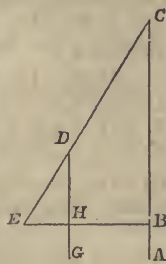


Suppose a man standing at A, desiring to know the distance to any object as B, on the opposite side of a river; how should he proceed to determine this distance, providing he has nothing but a ten-foot pole?

Form the right-angle BAC, and measure any distance, as AC = 30 feet. Then from C, measure towards A any distance, as CD = 3 feet; also DE, perpendicular to CA. Suppose DE=4 feet. Then by similar triangles we have

CD : CA : : DE : AB. Or,
3 : 30 : : 4 : 40, the distance AB.

The above principle is also employed in measuring the heights of trees, &c. In this case let AC be the height of the tree; E the height of the man's eye, which we will suppose = 5 feet; DG a perpendicular pole, and DH the height above the eye Then by similar triangles we have



EH : EB :: HD : BC. Suppose we have found the first three terms of this proportion to be as follows :—

3 : 24 :: 4 : CB. Then CB=32 ft. which being increased by AB=5 ft. we have 32+5=37 ft. the height of the tree.

Mensuration of Solids.

Problem 19.—Given the side of a cube, to find its solidity.

Multiply its length, breadth, and depth together.

1. What is the solidity of a cube, the side of which is 8 ft.?

Problem 20.—Given the length, and dimensions of the end of a prism, or parallelopipedon, to find its solidity.

Multiply the area of its base or end by its length.

1. What is the solidity of a triangular prism 24 feet long, and each side, $1\frac{1}{3}$ feet?
2. What is the solidity of a stick of timber 36 feet long and $2\frac{1}{2}$ feet square?

Problem 21.—Given the altitude and the base of a pyramid to find its solidity.

Multiply the area of its base by one third of its altitude.

1. What is the solidity of a triangular pyramid, the altitude of which is 24 feet, and each side of the base 6 ft.?

2. What is the solidity of a square pyramid, the altitude of which is 48 feet, and a side of the base 9 feet?

PROBLEM 22.—Given the altitude and the diameter of a cone, to find its solidity.

Multiply the area of its base by one-third of its altitude.

1. What is the solidity of a cone, the altitude of which is 16 feet, and the diameter of the base 3 feet?

2. What is the solidity of a cone, the altitude of which is 54 feet, and the diameter of the base 12 feet?

PROBLEM 23.—Given the altitude, and the dimensions of the two bases of a frustum of a pyramid, or of a cone, to find its solidity.

To the sum of the areas of the two bases, add the mean proportional between these two areas, and multiply the result by one-third of the altitude of the frustum.

REMARK.—A mean proportional between the areas of the two bases of a cone, is the square root of the product of the squares of the two diameters, into 0·7854.

1. What is the solidity of the frustum of a cone, whose altitude is 15 feet, and whose bases are 10 and 5 feet in diameter, respectively?

OPERATION.

$\overline{10}^2\times0{\cdot}7854=100\times0{\cdot}7854$, the area of the lower base.

$5^2\times0{\cdot}7854=25\times0{\cdot}7854$, the area of the upper base.

$\sqrt{100\times25}\times0{\cdot}7854=50\times0{\cdot}7854$, area of the mean proportional between the bases.

$175\times0{\cdot}7854$, the sum of the area of the three bases.

Hence,

$175\times0{\cdot}7854\times\frac{1}{3}$ of $15=687{\cdot}225$, cubic feet, the solidity of the cone.

REMARK.—This rule is of much importance in determining the solidity of round sticks of timber, the diameter of the ends of which differ.

2. There is a stick of timber, in the form of the frustum of a cone, that is 48 feet long, 4 feet in diameter at the larger end, and 9 inches at the smaller end. How many cubic feet does it contain?

ART. **263.** Another method of measuring round timber, is to

Multiply the square of one-fourth the girth, in inches, by the length of the stick, in feet; then divide the product by 144. *The quotient will be the contents in cubic feet.*

REMARK.—The girth should be taken two-thirds of the distance from the smaller to the larger end.

3. How many cubic feet in a stick of timber, 25 feet long, and whose girth is 60 inches?

4. How many cubic feet in a stick of timber 42 feet ong, and whose girth is 80 inches?

5. How many cubic feet in a stick of timber 36 feet long, and whose girth is 48 inches?

PROBLEM 24.—Given the diameter of a sphere, to find its solidity.

Multiply its surface by one-sixth of its diameter. Or,
Multiply the cube of the diameter by 0·5236.

1. What is the solidity of the Earth, supposing its diameter to be 8000 miles?

2. How many cubic inches in a cannon ball 9 inches in diameter?

It is believed that the foregoing rules will enable the pupil to solve most of the examples that may arise in ordinary mensuration.

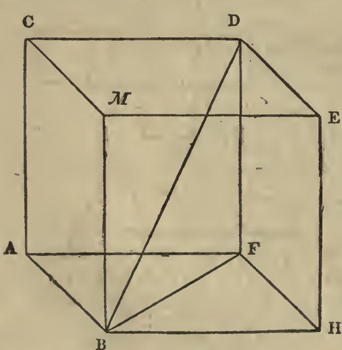
For the practical convenience of those who have occasion to refer to mensuration, we subjoin the following

TABLE OF MULTIPLES FOR MECHANICS.

1. Diameter of a circle × 3·1416 = Circumference.
2. Radius of a circle × 6·283185 = Circumference.
3. Square of the radius of a circle × 3·1416 = Area.
4. Square of the diameter of a circle × 0·7854 = Area.
5. Circumference of a circle × 0·159155 = Radius.
6. Square root of the area of a circle × 0·56419 = Radius.
7. Circumference of a circle × 0·31831 = Diameter.
8. Square root of the area of a circle × 1·12838 = Diameter.
9. Radius of a circle × 1·732051 = Side of inscribed equilateral triangle.
10. Diameter of a circle × 0·860254 = Side of inscribed equilateral triangle.
11. Side of inscribed equilateral triangle × 0·577350 = Radius of circle.
12. Radius of a circle × 1·414214 = Side of inscribed square.
13. Diameter of a circle × 0·7071 = Side of an inscribed square.

14. Side of inscribed square $\times$ 0·707107 = Radius.
15. Square of radius of a sphere $\times$ 12·566371 = Surface.
16. Square of the diameter of a sphere $\times$ 3·1416 = Surface.
17. Square of the circumference of a sphere $\times$ 0·3183 = Surface
18. Square root of surface of a sphere $\times$ 0·282095 = Radius.
19. Square root of the surface of a sphere $\times$ 0·56419 = Diameter
20. Square root of the surface of a sphere $\times$ 1·772454 = Circumference.
21. Cube of the diameter of a sphere $\times$ 0·5236 = Solidity.
22. Cube of the radius of a sphere $\times$ 4·1888 = Solidity.
23. Cube of the circumference of a sphere $\times$ 0·016887 = Solidity.
24. Cube root of solidity of a sphere $\times$ 0·6203505 = Radius.
25. Cube root of the solidity of a sphere $\times$ 1·2407 = Diameter.
26. Cube root of the solidity of a sphere $\times$ 3·8978 = Circumference.
27. Radius of a sphere $\times$ 1·1547 = Side of inscribed cube.
28. Side of inscribed cube $\times$ 0·8660254 = Radius.
29. The square of the side of a tetraedron $\times$ 1.7320508 = Surface.
30. The square of the side of a hexaedron $\times$ 6·0000000 = Surface.
31. The square of the side of an octaedron $\times$ 3·4641016 = Surface.
32. The square of the side of dodecaedron $\times$ 20·6457288 = Surface.
33. The square of the side of icosaedron $\times$ 8·6602540 = Surface.
34. The cube of the side of a tetraedron $\times$ 0·1178511 = Solidity.
35. The cube of the side of a hexaedron $\times$ 1·000000 = Solidity.
36. The cube of the side of an octaedron $\times$ 0·4714045 = Solidity.
37. The cube of the side of a dodecaedron $\times$ 7·6631189 = Solidity.
38. The cube of the side of an icosaedron $\times$ 2·181695 = Solidity.

A slight knowledge of the principles of geometry will enable the pupil to deduce the above multiplies. For illustration, determine the side of a cube that is inscribed in a sphere.



Let X = the side of the inscribed cube. In the right-angled triangle BFH, BF $= \sqrt{\overline{BH}^2 + \overline{FH}^2}$ (see Art. 240), or BF $= \sqrt{2\overline{BH}^2} = \sqrt{2X^2}$.

In the right-angled triangle, BDF, DF = X and BF $= \sqrt{2X^2}$; therefore, BD $= \sqrt{3X^2}$. But BD = the diameter of the sphere, which we will call *unity*; hence, $1 = \sqrt{3X^2}$, or $1 = X\sqrt{3}$ = X times 1·73205+; consequently, X, the side of the

cube, $=\frac{1}{1\cdot73205}$ of the diameter; or, $\cdot57736+$ times the diameter of the sphere; and since *Radius* equals $\frac{1}{2}$ of the diameter; X, the side of the cube, will equal *Radius* times $(2\times\cdot57736+)$ $=$ *Radius* times $1\cdot15472+$.

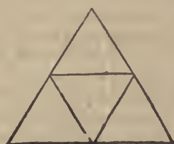
In a similar manner the pupil may deduce other multiples that he may desire, or the teacher require.

THE FIVE REGULAR BODIES.

ART. **264.** A regular body is a solid bounded by a certain number of similar and equal plane figures.

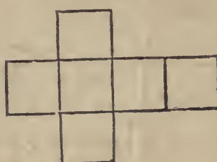
It is proved in Solid Geometry that only three kinds of equilateral and equiangular plane figures joined together can make a solid angle; hence but *five* regular bodies can possibly be formed.

1. A *Tetraedron* is a solid having four triangular faces.

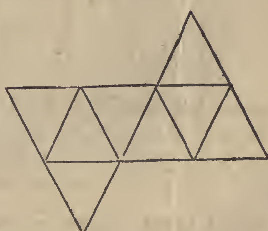


REMARK.—If figures similar to those annexed to the definitions, be drawn on pasteboard, and cut out, by cutting through the bounding lines, and if the other lines be cut half through, and then the parts be turned up and glued together, the bodies defined will be formed.

2. A *Hexaedron*, or cube, is a solid having six square faces.

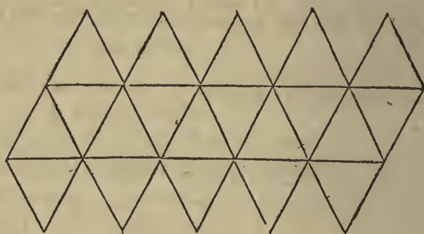


3. An *Octaedron* is a regular solid having eight triangular faces.



4. A *Doaecaedron* is a solid having twelve pentagonal faces.

5. An *Icosaedron* is a solid having twenty triangular faces.



SURFACES OF THE FIVE REGULAR BODIES.

PROBLEM 1.—Given the side of a *tetraedon*, to find its surface.

Multiply the square of the linear side by the $\sqrt{3}$.

1. If the side of a tetraedron is 1, what is its surface ?
2. If the side of a tetraedron is 8 feet, what is its surface ?

PROBLEM 2.—Given the side of a *hexaedron* to find its surface.

Multiply the square of the side by 6.

1. If the side of a hexaedron is 1, what is its surface ?
2. If the side of a hexaedron is 4 feet, what is its surface ?

PROBLEM 3.—Given the side of an *octaedron*, to find its surface.

Multiply the square of the side by $2\sqrt{3}$.

1. If the side of an octaedron is 1, what is its surface ?
2. If the side of an octaedron is 8, what is it surface ?

PROBLEM 4.—Given the side of a *dodecaedron*, to find its surface.

Multiply 15 *times the square of the side by* $\sqrt{1 + \frac{2}{5}\sqrt{5}}$.

1. If the lineal side of a dodecaedron is 1, what is its surface?

2. If the side of a dodecaedron is 9, what is its surface?

PROBLEM 5.—Given the side of an *icosaedron*, to find its surface.

Multiply 5 *times the square of the side by* $\sqrt{3}$.

1. If the side of an icosaedron is 1, what is its surface?

2. What is the surface of an icosaedron, the side of which is 6 feet?

REMARK.—From the answers of the examples given under the preceding problems, in which the lineal side is 1, the following table may be formed.

TABLE

Showing the surfaces of the five regular bodies, when the linear side is 1.

Number of sides.	Names of bodies.	Surfaces of bodies.
4	Tetraedron.	1·7320508
6	Hexaedron.	6·0000000
8	Octaedron.	3·4641016
12	Dodecaedron.	20·6457288
20	Icosaedron.	8·6602540

PROBLEM 6.—Given the side of any of the regular bodies, to find its surface.

Multiply the square of the length of the side by the tabular area opposite the figure mentioned.

1. The side of a tetraedron is 14 in.; what is its surface?

2. The side of a hexaedron is 7 feet; what is its surface?

3. The side of a tetraedron is 18 feet; what is its surface?

4. The side of an octaedron is 12 inches; what is its surface?

5. The side of a hexaedron is 16 feet; what is its surface?

6. The side of a dodecaedron is 18 inches; what is its surface?

7. The side of an octaedron is 10 feet; what is its surface?

8. The side of an icosaedron is 20 inches; what is its surface?

9. The side of a dodecaedron is 24 feet; what is its surface?

10. The side of an icosaedron is 16 feet; what is its surface?

SOLIDITY OF THE REGULAR BODIES.

PROBLEM 1.—Given the side of a tetraedron, to find its solidity.

Multiply $\frac{1}{12}$ *of the cube of the lineal side by the* $\sqrt{2}$.

1. If the lineal side of a tetraedron is 1, what is its solidity?

2. If the side of a tetraedron is 4 inches, what is its solidity?

PROBLEM 2.—Given the lineal side of a hexaedron, to find its solidity.

Cube the Side.

1. If the lineal side of a hexaedron is 1, what is its solidity?

2. If the side of a hexaedron is 6 feet, what is its solidity?

PROBLEM 3.—Given the side of an octaedron, to find its solidity.

Multiply the cube of the side by the $\sqrt{2}$, *and* $\frac{1}{3}$ *of the product will be the solidity.*

1. What is the solidity of an octaedron, the side of which is 1?

2. What is the solidity of an octaedron, the side of which is 4 inches?

PROBLEM 4.—Given the side of a dodecaedron, to find its solidity.

Add 47 *to* $21\sqrt{5}$, *and divide this sum by* 40; *then multiply the square root of this quotient by* 5 *times the cube of the side.*

1. What is the solidity of a dodecaedron, the side of which is 1 ?

2. What is the solidity of a dodecaedron, the side of which is 8 inches ?

PROBLEM 5.—Given the side of an icosaedron, to find its solidity.

Divide the sum of 7 *and* $3\sqrt{5}$ *by* 2; *then multiply the square root of this quotient by* $\frac{5}{6}$ *of the cube of the side.*

1. What is the solidity of an icosaedron, the side of which is 1 ?

2. What is the solidity of an icosaedron, the side of which is 4 inches ?

REMARK.—From the answers of the examples given under the preceding problems, in which the lineal side is 1, the following table may be formed.

TABLE.

Number of Sides.	Names of Bodies.	Solidity of Bodies.
4	Tetraedron.	.1178511
6	Hexaedron.	1.0000000
8	Octaedron.	.4714045
12	Dodecaedron.	7.6631189
20	Icosaedron.	2.1816950

PROBLEM 6.—Given the side of any of the regular bodies, to find its solidity.

Multiply the cube of the side by the solidity opposite the given figure in the above table.

1. What is the solidity of the five regular bodies, respectively, the side of each being 8 inches ?

2. What is the solidity of the five regular bodies, respectively, the side of each being 12 inches.

PHILOSOPHICAL PROBLEMS.

ART. **265.** *The spaces described by bodies falling from a state of rest under the influence of gravity, are propor-*

tioned to the SQUARES OF THE TIMES *during which they are falling.**

ART. **266.** *The spaces described by falling bodies are also proportioned to the* SQUARES OF THE *velocities which they acquire in falling over those spaces.**

A body in 1 second of time will fall $16\frac{1}{12}$ feet; call it **16** *feet.* The velocity acquired in the same time is **32** *feet.*

Let T equal the time, during which a body has been falling; D, the distance it has fallen; and V, the velocity acquired.

From the above *laws* and *facts* we have the following *proportions*, from which the general formulas relating to falling bodies may be deduced.

1. **2.**

$$1^2 : T^2 :: 16 : D; \text{ hence, } D=16T^2; \therefore, T=\sqrt{\frac{D}{16}}=\tfrac{1}{4}\sqrt{D}$$

3. **4.**

$$\overline{32}^2 : V^2 :: 16 : D; \text{ hence, } D=\frac{V^2}{64}; \therefore, V=\sqrt{64D}=8\sqrt{D}$$

Placing the right hand member of **1** and **3** equal to each other, we have,

$16\ T^2 = \frac{V^2}{64}$;—From which we deduce the following :

$$\textbf{5.}\quad T=\frac{V}{32}$$

$$\textbf{6.}\quad V=32T$$

Pupils should become familiar with the six preceding formulas, which we will arrange differently for the convenience of reference.

1. $T=\frac{1}{4}\sqrt{D}$	4. $D=\frac{V^2}{64}$
2. $T=\frac{V}{32}$	5. $V=8\sqrt{D}$
3. $D=16T^2$	6. $V=32T$

* Olmsted's Natural Philosophy.—*However, these laws are not strictly correct.*

1. How far will a leaden ball fall in 12 seconds; 14 seconds; 25 seconds; and 60 seconds, respectively? (See formula 3d.)

2. How long will a body be in falling 1024 feet; 1600 feet; 10000 feet; and 722500 feet, respectively? (See formula 1st.)

3. In what time would a body acquire a velocity of 128 feet; 160 feet; 288 feet; 1024 feet; and 3072 feet, respectively? (See formula 2nd.)

4. What velocity would a body acquire in 4; 7; 9; 12; 25; and 60 seconds, respectively? (See formula 6th.)

5. What velocity would a body acquire in falling 1024 feet; 7225 feet; 625 feet; 3025 feet; and 9025 feet, respectively? (See formula 5th.)

6. Through what space would a body have fallen to acquire a velocity of 96 feet; 192 feet; 768 feet; 288 feet; and 384 feet, respectively? (See formula 4th.)

ART. **267.** *The time of the vibrations of pendulums are to each other as the square roots of their lengths; hence, their lengths are as the squares of their times of vibration.*

A pendulum that vibrates seconds is $39\frac{1}{2}$ inches in length.

1. What is the length of a pendulum that shall vibrate 3 times a second?

2. What is the length of a pendulum that shall vibrate once in 5 seconds?

3. What is the length of a pendulum that shall vibrate once in a minute?

4. How often will a pendulum vibrate, the length of which is 225 inches?

5. How often will a pendulum vibrate, the length of which is 144 feet?

ART. **268.** The gravity of any body above the earth's surface *decreases*, as the squares of its distance, in semi-diameters of the earth, from its centre *increases*. Hence,

The weight of a body on the earth's surface, is to its weight at any assignable distance above the surface of the

earth; as the square of its distance from the earth's centre, to the square of the earth's semidiameter, and vice versâ.

1. If a body weigh 750 pounds at the earth's surface, how much would it weigh 20000 miles above its surface?

2. If a body at the earth's surface weigh 3600 pounds, how much would it weigh 240000 miles above its centre, the distance of the moon from the earth?

If a body at the earth's surface weighed 1800 pounds but being carried to a certain height weighs only 200 pounds, what is that height?

MISCELLANEOUS QUESTIONS.

REMARK.—ANALYSIS has been so extensively treated of in the "American Intellectual" and the "Practical Arithmetic," that it is deemed unnecessary to add anything more to what has already been given in the preceding pages.

1. A gentleman, after losing $\frac{3}{7}$ of all his money, had \$368 remaining. How much had he at first?

2. Thomas has 364 sheep more than James, and they together have 1588. How many have they respectively?

3. Henry, Perry, and John, found a purse containing \$768, which they agree to share in proportion to the numbers 3, 4, and 5. How much should each receive?

4. A farmer gave to a certain number of laborers, \$14 apiece, if he had given them \$19 apiece it would have taken \$125 more. How many laborers were there?

5. A fish-pole, the length of which was 24 feet, was broken into two pieces; and $\frac{3}{5}$ of the length of the longer piece equalled the length of the shorter. What was the length of each piece?

6. There is a fish whose head is 18 inches long, and whose tail is as long as its head + $\frac{2}{3}$ of the length of its body, and whose body is as long as its head and tail both. What is the length of the fish?

7. Henry is 18 years old, and Harvey is 14; how many years since was Henry twice as old as Harvey?

8. A and B, together, have \$8645, but A has \$155 more than 3 times as much as B. How many dollars has each?

9. A gentleman bought a hat, a vest, and a pair of pants. The hat cost \$6. The hat and vest cost *twice* as much as the pants, and the hat and pants cost 3 times as much as the vest. What was the cost of the vest and pants, respectively?

10. A can earn a certain sum of money in 20 days; A and B together, can earn the same sum in 6 days. How long will it take B alone to earn the same sum?

11. A merchant bought a certain number of yards of cloth for \$245, and after using 9 yards of it himself, sold $\frac{3}{8}$ of the remainder for \$99, which was \$24 more than it cost. How many yards did he buy at first?

12. A person at a game of cards lost $\frac{3}{4}$ of all his money, and then won \$144; he now lost $\frac{3}{4}$ of all the money he had, and found he had but \$95 remaining. How much had he at first?

13. There is a cask containing brandy and water; $\frac{2}{5}$ of the whole, + 12 gallons is water; and $\frac{3}{7}$ of the whole + 8 gallons is brandy. How many gallons of each?

14. Two brothers, James and Henry, have the same income. James contracts an annual debt, amounting to \$165; Henry lives on $\frac{5}{7}$ of his income, and saves yearly \$101, after lending James enough to pay his debt. How much was the yearly income of each?

15. Two masons, A and B, together can do a certain piece of work in 10 days; how long would it take each separately to do it providing A does 3 times as much as B?

16. If A and B can, together, do a certain piece of work in 5 days; A and C, in 6 days; and B and C, in $7\frac{1}{2}$; how many days would it require for each to perform the work alone?

17. Divide \$4760 among three persons, James, William and Mary, so that James' part shall be to William's as 2 to 3, and that Mary shall have as much as James and William together lacking \$920.

18. A and B can, together, do a certain piece of work in 10 dys.; A and C, in 15 dys.; and B and C, in 20 dys In how many days could each perform the work alone?

19. A gentleman distributed a certain number of dollars among four poor women in the following manner:—to the first he gave $\frac{1}{2}$ of the number of dollars he had + \$$\frac{1}{2}$; to the second $\frac{1}{2}$ the remainder + \$$\frac{1}{2}$; in the same manner he gave to the third and the fourth; and found he had yet one dollar remaining. How many dollars had he at first, and how much did he give to each woman?

20. An estate of \$12850 was was left to four brothers, who are 17, 15, 13, and 9 years of age, to be so divided that the respective parts, being placed out at 5 per cent. simple interest, should amount to equal sums when they become 21 years of age, respectively. How much was each one's share?

21. A man bought a horse for \$102, which was $\frac{3}{7}$ of twice as much as he sold it for, lacking \$2. How much did he gain by the bargain?

22. A woman bought 60 oranges. For $\frac{3}{5}$ of them she paid 5 cents for 3 oranges; and for the remainder 3 cents for five; for how much must she sell them apiece to gain $33\frac{1}{3}$ per cent.?

23. A farmer paid to four of his hired men $77\frac{1}{5}$ bushels of wheat. The first earned 1 bushel as often as the other three earned $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ of a bushel, respectively. How many bushels should each receive?

24. Two men A and B were playing cards; B lost \$84, which was $\frac{7}{10}$ times $\frac{2}{3}$ as much as A then had. When they commenced playing, $\frac{5}{8}$ of A's money equalled $\frac{3}{5}$ of B's; how much had each when they began to play?

25. What is the interest on \$685·95 from April 14th to Sept. 19th?

26. What is the amount of \$684·99 from April 9th, 1853, to July 8th, 1854?

27. A has with B the following account:—

1853.	*Dr.*	1853.	*Cr.*
March 12th, Due .	\$845·45	Sept. 16th, Due. .	\$784·50

At what time is the balance of the account due?

28. I sold the following bills of goods, on the conditions below stated:

March 6, 1853,	a bill	amounting to	$480 on 4	months'	credit.
April 15,	"	"	"	$670 on 6	"	"
May 25,	"	"	"	$785 on 4	"	"
June 28,	"	"	"	$670 on 3	"	"

How much money will balance the account July 20th ?

29. Three farmers, A, B, and C, together have 1920 acres of land; A has 40 acres more than B; and C has as many as A and B together, lacking 32. How many acres has each ?

30. What is the discount on $847·50 from May 12th, 1852, to July 25th, 1854 ?

31. What sum of money will give $184·90 interest from June, 16th, 1853, to Sept. 18th, 1854 ?

32. If A can do a certain piece of work in 80 days, and with the assistance of C, in $34\frac{2}{7}$ days; how long will it take C to do the work alone ?

33. Three carpenters, A, B, and C, earn a certain sum of money in 24 days; A and B can earn the same amount in 48 days; and A and C, in 36 days. How long would it take each separately to earn the same amount ?

34. An individual being requested to buy a certain number of pounds of meat, found, if he bought beef, at $11\frac{1}{2}$ cts. a pound, he would have 90 cts. remaining; but if he bought pork, at $17\frac{1}{3}$ cts. a pound, he would lack 15 cts. of having money enough to pay for it. How many pounds of meat was he requested to buy, and how much money did he have ?

35. A lady bought a certain number of apples, at the rate of 5 for 2 cents; and paid for them with oranges, at the rate of 3 for 2 cents. How many apples did she buy, providing it took 144 oranges to pay for them ?

36. Three farmers, Thomas, William, and Henry, talking of their sheep; says Thomas to William, I have 4 times as many sheep as you; says William to Henry, I have $\frac{3}{5}$ as many as you; and says Henry to Thomas, if I had 63 sheep more, I should then have as many as you. How many had each ?

37. A man was hired for 160 days, on this condition: that for every day he worked, he should receive $1·44, and for every day he was idle, he should pay 96 cents for

his board. At the expiration of the time he received $64 How many days did he work?

38 A, B, and C formed a co-partnership: A advanced $10000; B $8000; and C $7000. At the end of 6 months, A withdrew $3000 from the business; B withdrew $1500; and C increased his stock by $\frac{2}{7}$ of its original amount. At the end of the year, they had gained $5584·60. How much should each receive?

39. A gentleman willed $8640 to his wife, son, and daughter, to be divided among them in the proportion of $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{1}{2}$. The widow dying soon after, the whole sum was divided in due proportion between the two children. How much did each receive?

40. A cistern receives water from 3 pipes; the first of which would fill it in 12 hours; the second in 8 hours; and the third in 6 hours. In what time would these three pipes together fill the cistern, providing $\frac{1}{8}$ of the whole capacity of the cistern leaked out in each hour?

41. A merchant spent $\frac{2}{5}$ of his money for silks; $\frac{2}{3}$ of the remainder for dry goods; $\frac{3}{4}$ of the remainder for groceries; and the remainder, which was $287·65, for stationery. How much money did he expend in all?

42. Four men contracted to grade a turnpike road for $12000. In accomplishing the work, one of the men furnished 45 laborers for 74 days; another, 54 laborers for 66 days; another, 75 laborers for 84 days; and the other, 95 laborers for 85 days. How much should each contractor receive?

43. An agent receives $5685 to invest in merchandise, at a commission of $1\frac{2}{3}$ per cent. on the amount of purchase that can be made after his percentum is deducted. What is the amount of purchase; also, his commission?

44. An upholsterer realized a profit of 25 per cent. by selling carpeting, at $1.50 a yard. What would have been the loss per cent. if he had sold it at $0·80 a yard?

45. How much would a person gain or lose by borrowing $2000 from May 12th, 1852, to Nov. 12th, 1854, at 7 per cent, and lending the same sum, at $6\frac{1}{2}$ per cent.,

and on such conditions as will enable him to compound the interest every 6 months?

46. A drover bought 288 head of cattle, at $\$42\frac{3}{4}$ a head, and pays for them with the proceeds of a note which is discounted in a bank for 90 days, at 7 per cent. At the end of 25 days, he sells the cattle, at $\$68\frac{1}{2}$ a head, and puts the proceeds on interest, at $8\frac{3}{4}$ per cent., until his note is to be paid at the bank. What profit does he make by these transactions, after paying $374·65 for the cattle while he had them?

47. A merchant took a farmer's note for $585·50, due, without interest, May 14th, 1853. Some time afterwards, the farmer got possession of a note against the merchant for $894·85, due, without interest, Nov. 25th. When, in equity, ought the balance to be paid? Suppose money to be worth 7 per cent., and they desire to settle Aug. 15th; how stands the matter of debt between them?

48. A is indebted to B $885; $125 of which is due May 4th; $244, June 18th; $345, Aug. 12th; and the remainder, Oct. 25th,—without interest. At what time might the whole, in equity, be paid at once?

49. What must be the dimensions of a granary which shall contain 2400 bushels of wheat; its length to be twice its breadth, and its breadth and height equal?

50. What is the difference in *area* between two fields of the same perimeter; one of which is a square, and the other 85 rds. long, and $25\frac{1}{2}$ rods wide?

51. An individual was requested to purchase 1084 bushels of grain, consisting of rye, wheat, and barley; $\frac{3}{4}$ of the number of bushels of rye was to equal $\frac{2}{3}$ of the number of bushels of wheat, and $\frac{3}{4}$ of the number of bushels of wheat was to equal $\frac{2}{5}$ of the number of bushels of barley. How many bushels of each kind must he buy?

52. A man being asked the hour of the day, replied, that $\frac{3}{5}$ of the time past noon equalled $\frac{3}{4}$ of the time from now to midnight. What was the time?

53. A tree, whose length was 156 feet, was broken into two pieces by falling; $1\frac{1}{2}$ times the length of the top piece,

equals $1\frac{1}{8}$ the bottom piece, + 12 feet. What is the length of the two pieces respectively?

54. A man bought a cow, a horse, and an ox for $350. For the horse he gave 4 times as much as for the ox, lacking $40; and for the ox, twice as much as for the cow, lacking $12. What did he give for each?

55. A farmer has 299 sheep in two different fields; the number in the first field equals $1\frac{3}{4}$ times the number in the second field, + 48. How many are there in each field?

56. An individual, after spending $\frac{3}{4}$ of all his money, and $\frac{4}{5}$ of what remained, lacking 12\frac{1}{2}$, had only 347\frac{1}{2}$ remaining. How much had he at first?

57. There is an island 36 miles in circumference, and three men, A, B, and C start from the same point, and travel the same way around it; A 4 miles an hour; B, 12; and C, 20. In what time will they all be together; and in what time will they all meet at the place from which they started?

58. A note of $1200, given Feb. 3d., 1851, has received the following indorsements:- March 12th, 1852, indorsed $365·45; Nov. 14th, 1852, indorsed $285·90; Jan. 12th, 1853, indorsed $484·12$\frac{1}{2}$. How much remains due March 20th, 1854, interest computed at 7 per cent.?

59. Four masons, A, B, C, and D engage to build a certain piece of wall for $660. While A can build 5 rods, B can build $7\frac{1}{2}$, C $3\frac{3}{4}$, and D $6\frac{1}{4}$. When the wall is $\frac{2}{3}$ completed, D ceases to labor upon it, and A, B, and C finish it. How much should each receive?

60. A market-woman bought oranges, at 10 cents a dozen, half of which she exchanged for lemons, at the rate of 9 oranges for 7 lemons; she then sold all her oranges and lemons, at $1\frac{2}{3}$ cents apiece, and thereby gained 24 cents. How many oranges did she buy, and how much did they cost?

61. If A can perform a certain piece of work in $\frac{2}{5}$ of a day; B $\frac{1}{8}$ of a day; and C in $\frac{1}{10}$ of a day; how many times longer will it take C to do the work alone, than it will take A and B together to do it?

62. A traveler had stolen from him $\frac{13}{17}$ of all his money:

the thief was caught, but not until he had spent $\frac{3}{4}$ of it, the remainder, \$647·37$\frac{1}{2}$, was given back. How much money had the traveler at first?

63. Three men, A, B, and C, built a stone wall: A built 15 rods; B built as much as A; $\frac{3}{5}$ as much as C; and C built as much as A and B together, lacking 5 rods. How many rods did they all build, and how many did B and C build, respectively?

64. At what time between 2 and 3 o'clock will the hour and minute hands of a clock be together?

65. A person being asked his age, replied, that if his age were increased by its $\frac{2}{5}$, its $\frac{5}{6}$, and 25$\frac{1}{2}$ years more, the sum would equal 3$\frac{1}{2}$ times his age. What was his age?

66. A person being asked the time of day, replied, that $\frac{3}{4}$ of the time past noon, equal $\frac{3}{5}$ of the time from then to midnight, lacking 12 minutes and 36 seconds. What was the time?

67. When James was married, he was 3 times as old as his wife, but when they had been married 60 years, he was only 1$\frac{2}{3}$ times as old. How old was each when they were married?

68. Four individuals found a purse, containing \$2445, which they agree to share in the proportion of $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{1}{2}$. How much should each receive?

69. A deer is a 180 leaps before a hound, and takes 4 leaps to the hound's 9; and 5 of the deer's leaps are equal to 9 of the hound's. How many leaps must the hound take to catch the deer?

70. A market-woman bought a certain number of pine-apples, at the rate of 3 for 40 cents, and as many more at the rate of 5 for 45 cents; and sold them all at the rate of 7 for 87$\frac{1}{2}$ cents, and thereby gained \$1·60. Hcw many pine-apples did she buy?

71. A boy bought a certain number of apples at the rate of 4 for a cent, and as many more at 5 for a cent; and sold them out, at the rate of 9 for 5 cents, and by so doing gained 45 cents. How many apples did he buy?

72. A mechanic and his two sons earned \$1490 in 1 year; the father earned twice as much as the elder son,

lacking \$70, and the younger son earned $\frac{1}{3}$ as much as the elder son + 160 dollars. How much did each earn?

73. A woman bought a certain number of oranges, at the rate of 5 for 3 cents, as many more at the rate of 7 for 5 cents; and sold them all, at the rate of 15 for 11 cents, and thereby gained 25 cents. How many oranges did she buy?

74. A merchant bought three pieces of cloth for \$639: $\frac{3}{4}$ of the cost of the first piece equals $\frac{6}{7}$ of the cost of the second; and $\frac{4}{5}$ of the cost of the second piece equals $\frac{8}{9}$ of the cost of the third. How much did each piece cost?

75. A merchant bought three pieces of cloth; the first piece contained $\frac{3}{5}$ as much as the second piece + 12 yards; and $\frac{3}{4}$ of the number of yards in the second piece equaled $\frac{3}{5}$ of the number of yards in the third. How many yards in each piece, providing there were 8 yards more in the third piece than in the second?

76. It is found that $\frac{2}{5}$ of A's + $\frac{4}{9}$ of B's fortune equals \$5400; and that $\frac{3}{4}$ of A's fortune equals $1\frac{2}{5}$ times $\frac{3}{7}$ of B's + \$24. What is the fortune of each?

77. A hound ran 150 rods before he caught a hare; and $\frac{2}{13}$ the distance the hare ran before it was caught equaled the distance it was a-head when they started. How far after the chase commenced, did the hare run before it was caught?

78. A and B started from the same point, and ran in the same direction; B ran 132 rods; then $\frac{2}{13}$ the distance A had run equaled the distance A was in advance of B. How much did A gain on B in running 132 rods?

79. A gentleman left his son a fortune; $\frac{1}{7}$ of which he spent in 2 years; $\frac{1}{3}$ of the remainder lasted him 3 years longer; $\frac{3}{4}$ of the remainder lasted him 5 years longer when he had only \$7849·12$\frac{1}{2}$ left. How much did his father leave him?

80. $\frac{2}{3}$ of A's number of sheep is to $\frac{3}{4}$ of B as $\frac{3}{4}$ to $\frac{6}{7}$; and $\frac{3}{4}$ of B's number + $\frac{2}{3}$ of A's equals 360. How many sheep has each?

81. Find the fortunes of A, B, C, D, E and F, by knowing that B is worth \$220, which is $\frac{1}{4}$ as much as A and C

are worth, and that A is worth $\frac{1}{3}$ as much as B and C; and also, that, if 76 times the sum of A's, B's, and C's fortune were divided in the proportion of $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{3}$, it would, respectively, give $\frac{2}{3}$ of D's, $\frac{4}{5}$ of E's, and $\frac{5}{6}$ of F's fortune.

82. There is a park 16 rods square, and it is desired to make a gravel walk around it that shall contain $\frac{15}{64}$ of the whole area of the park. What should be the width of the gravel walk?

83. A speculator sold flour at \$5 a barrel; $\frac{1}{6}$ of which equaled his gain. How much would he have gained per cent. if he had sold it at \$6·25 a barrel?

84. A merchant sold a quantity of goods for \$6784; and thereby cleared $\frac{1}{12}$ of this money. If he had sold them for \$6999, what would he have gained per cent.?

85. A speculator sold a quantity of cotton for \$8484; and by so doing gained $\frac{1}{6}$ of what it cost him. How much would he have gained per cent. if he had sold it for \$9898?

86. A gentleman bought $\frac{3}{4}$ of a farm for \$9000; and sold to B $\frac{1}{2}$ of his share; B sold to C $\frac{2}{3}$ of what he received; C sold to D $\frac{3}{5}$ of what he received; and D sold to E $\frac{5}{8}$ of what he received. What part of the farm did each man buy, and how much did it cost him?

87. I bought $\frac{3}{4}$ of a house, valued at \$18000; and sold $\frac{1}{2}$ of my share to A; A sold $\frac{1}{3}$ of his share to B; and B sold $\frac{2}{5}$ of his share to C. What part of the value of the house does each own, and how much does C pay for his part?

88. An individual sold two horses, at \$630 apiece; for one he received 25 per cent. more than its value, and for the other 25 per cent. less than its value. Did he gain or lose by the bargain, and how much?

89. B's fortune added to $\frac{2}{3}$ of A's, which is to B's as 2 to 3, being put on interest for 6 years, at 8 per cent., amounts to \$988. What is the fortune of each?

90. How much grain must a farmer take to mill, that he may fetch away 14·4 bushels, after the miller has taken $7\frac{9}{13}$ per cent. of all he took there?

91. The interest on the sum of $\frac{1}{2}$ of A's + $\frac{2}{3}$ of B's

money for 4 years, at 6 per cent., is \$480. What is the fortune of each, providing $\frac{1}{2}$ of B's money equals 3 times $\frac{2}{3}$ of A's?

92. The amount of $\frac{3}{5}$ of A's fortune + $\frac{4}{5}$ of B's for two years, at 5 per cent., is \$4950. What is the fortune of each, providing $\frac{3}{5}$ of A's money equals only $\frac{2}{7}$ of $\frac{4}{5}$ of B's?

93. If the interest on the sum of A's and B's fortune for 7 years and 6 months, at 4 per cent., is \$3213; and $\frac{3}{4}$ of A's fortune equals $\frac{2}{3}$ of B's; what is the fortune of each?

94. What will be the result, if from the sum of 3, $\frac{3}{4}$ $3\frac{1}{2}$, $\frac{3}{4}$ of 3, $3\frac{1}{2}$ of $3\frac{1}{4}$, $\frac{5}{7}$ of $\frac{3\frac{1}{2}}{2\frac{1}{2}}$ we subtract the sum of $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{1}{3}$, $\frac{1}{4}$ of $3\frac{1}{2}$; $\frac{2\frac{1}{2}}{3}$ of 5, $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ of $\frac{3\frac{1}{3}}{2\frac{1}{2}}$, and 3 $\frac{1}{8}$; multiply this difference by the greatest common divisor of 315 and 405; divide this product by the least common multiple of 6, 9, and 24; reduce the quotient to its lowest terms; add $\frac{1}{3}$ of $\frac{3}{4}$ to the result; multiply $\frac{3}{4}$ of this sum by $2\frac{1}{2}$; and divide the product by $\frac{1}{2}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ of $\frac{32}{45}$ of $\frac{2\frac{1}{2}}{2\frac{1}{3}}$?

95. Divide \$3106·50 among A, B, C, and D, in the following proportion:—A, B, and C are to have $\frac{47}{60}$ of it; B, C, and D are to have $\frac{37}{60}$ of it; A, C, and D are to have $\frac{7}{10}$ of it; and A, B, and D are to have $\frac{3}{4}$ of it. According to the above estimates, how much ought each to receive?

96. An individual, for two successive years, spent $\frac{1}{6}$ more than his yearly income; and found that, in 6 years, by saving $\frac{1}{20}$ of his annual income, he was able to discharge the debt, and have \$80 remaining. What was his annual income?

97. How many cannon balls, 8 inches in diameter, can be put into a cubical vessel, 2 feet on a side; and how many gallons of wine will it contain after it is filled with balls, allowing the balls to be hollow, the hollow being 4 inches in diameter, and the opening leading to it, to contain $1\frac{1}{2}$ solid inches?

98. A farmer sold hay, at \$10·50 a ton, and cleared $\frac{1}{3}$ of his money; but hay growing scarce, he sold it, at \$12 a ton. What did he clear per cent. by the latter price?

99. If 24 men, in 132 days of 9 hours each, dig a trench that is 4 degrees of hardness, $337\frac{1}{2}$ feet long, $5\frac{3}{5}$ feet wide, and $3\frac{1}{2}$ feet deep; how many men will be required to dig a trench that is 7 degrees of hardness, $232\frac{1}{2}$ feet long, $3\frac{2}{3}$ feet wide, and $2\frac{1}{3}$ feet deep, in $5\frac{1}{2}$ days of 11 hours each?

100. From a certain sum of money I took away its $\frac{1}{2}$, and in its stead placed \$200; I then took from this sum its $\frac{1}{4}$, and in its stead placed \$100; I now took away its $\frac{1}{3}$, and found I had only \$480 left. How much was the original sum?

101. From a sum money, \$360 more than its $\frac{1}{5}$ was taken away; from the remainder, \$280 more than its $\frac{1}{3}$ was taken away; and from what now remained, \$80 more than its $\frac{2}{5}$ was taken away, and then there remained only \$80. What was the original sum?

102. From a certain sum of money I took its $\frac{1}{2}$, and put in its stead \$460; from the remainder I took its $\frac{2}{5}$, and put in its stead \$600; and from what then remained I took its $\frac{1}{3}$, and put in its stead \$840, and found I had twice as much money as I had at first. How much had I at first?

103. Make the sum, difference, product, and quotient of 15 and 45 the numerators of fractions which shall have 75, 40, 750, and 60 for denominators; reduce them to equivalent fractions having a common denominator; subtract the sum of the last two fractions from the sum of the first two; multiply this difference by the first fraction; divide the product by the greatest common divisor of the numerators; multiply the quotient by the least common multiple of the denominators; add the first fraction reduced to a decimal to this quotient; subtract the second fraction reduced to a decimal from this sum; multiply this remainder by the third fraction reduced to a decimal; divide this product by the fourth reduced to a decimal;

then reduce the quotient to a vulgar fraction. What is the result?

104. A merchant sold 3 pieces of broadcloth, each piece containing 27 yards, at $7 a yard, on 2 months' credit, and made 12 per cent. on the first cost,—it had been on hand 3 months; 7 pipes of wine, at $4·50 per gallon, at an advance of 18 per cent. on the first cost, which had been 7 months on hand,—for which he gave a credit of 3 months; and 7 bales of cotton, at 11½ cents a pound, each bale containing 230 pounds, which had been on hand 1 month and 15 days, at an advance of 20 per cent. on the first cost,—for which he gave 6 months credit. How much did he make by the operation, and how much did he make on each article?

105. Suppose premiums, of three grades, to the amount of $24 are to be distributed among the pupils of a school. The value of a premium of the first grade is twice the value of one of the second grade; the value of one of the second grade is twice the value of one of the third grade; and there are 6 of the first grade, 12 of the second, and 6 of the third. What is the value of a premium of each grade?

106. Four carpenters built a house in company. The lot on which they built it cost $1000; the lumber and building materials of all kinds cost $6500; they paid for mason-work $500; and for painting and glazing $350. Of these expenses A paid $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D the residue. A worked on the house 45 days, at $1·50 a day, with 3 apprentices, each $0·75 a day; B worked 75 days, at $1·75 a day, with 2 journeyman, each $1·25 a day; C worked 60 days, at $1·62½ a day, with 1 journeyman, at $1·37½ a day, and 2 apprentices, each $0·87½ a day; and D, the master workman, worked 90 days, at $2·25 a day, with 2 journeyman, each $1·75 a day, and 2 apprentices, each $1·25 a day. The house being completed it was sold for $2500 more than it cost. How much in equity ought each partner to receive?

107. A deer starts 40 rods before a hound, and is not perceived by him until 40 seconds afterwards; the deer runs, at the rate of 10 miles an hour; and the hound after

it, at the rate of 18 miles an hour. What distance will the hound run before he overtakes the deer, and how long will the chase continue?

108. Two men in New York hired a carriage for $25, to go to New Haven, a distance of 72 miles, and return, with the privilege of taking in three more persons. Having gone 20 miles, they take in A; at New Haven they take in B; and when within 30 miles of New York they take in C. How much in equity ought each man to pay?

109. A boy went to a store and spent $\frac{1}{2}$ his money, and $\frac{1}{2}$ of a cent more for pine-apples; he then went to another store and spent $\frac{1}{2}$ the money he had remaining, and $\frac{1}{2}$ of a cent more for oranges; he now went to a third store and spent half the money he had remaining, and $\frac{1}{2}$ of a cent more for lemons; and then had only 9 cents remaining. How much money had he at first, and how much did he expend for pine-apples, oranges, and lemons respectively?

110 A father left his four sons, whose ages are 15, 11, 8, and 6 years respectively, $5777, to be so divided that the respective parts being placed out, at 6 per cent. simple interest, shall amount to equal sums when they become 21 years of age. What are these parts?

111. An individual, at a public-house borrowed as much money as he had, and spent $12\frac{1}{2}$ cents; he then went to another, where he borrowed as much money as he then had, and spent $12\frac{1}{2}$ cents; then went to a third, and a fourth and did the same; and then had no money remaining. How much money had he at first?

112. An estate of $17768 is to be divided among a widow, two sons, and two daughters, so that each son shall receive twice as much as each daughter, lacking $240; and the widow as much as all the children, lacking $520. What was the share of each?

113. A, B, and C can perform a certain piece of work in 24 days; how long will it take each to perform the work alone, if A does $1\frac{2}{3}$ times as much as B, and B does $\frac{2}{3}$ as much as C?

114. A farmer, having sheep in two different fields, sold $\frac{1}{5}$ of the number from each field, and had only 280

sheep remaining. Now 20 sheep jumped from the first field into the second; then the number remaining in the first field was to the number remaining in the second field as 5 to 9. How many sheep were there in each field at first?

115. A farmer paid five laborers a certain sum of money every month; to the first he paid $\frac{1}{2}$ the whole sum, lacking \$16; to the second $\frac{1}{2}$ of the remainder, lacking \$8; the third $\frac{1}{2}$ of the remainder, lacking \$4; to the fourth $\frac{1}{2}$ of the remainder, lacking \$2; and to the fifth the remainder, which was \$11. How much did he give them all a month, and how much to each?

116. A Californian on his way home with \$4000, was met by a party that robbed him of $\frac{2}{3}$ of $\frac{5}{8}$ of all he had; a second party met and robbed him of $\frac{2}{5}$ of $\frac{5}{8}$ of the remainder; a third party met him and robbed him of $\frac{7}{10}$ of $\frac{17}{20}$ of what he had left; and a fourth party took from him $\frac{1}{4}$ of $\frac{3}{4}$ of what still remained. How much money had he left?

117. A gentleman promised his son a new arithmetic, if he would go to a certain orchard, which was entered through three gates, and get such a number of apples, that, on his return, he could leave at the first gate, $\frac{1}{2}$ the apples he had and $\frac{1}{2}$ an apple more; at the second gate, $\frac{1}{2}$ of what he had remaining and $\frac{1}{2}$ an apple more; and at the third gate, $\frac{1}{2}$ the apples he still had remaining, and $\frac{1}{2}$ an apple more, without cutting any; and then have 17 apples remaining. How many apples must he get, and how many will he leave at the gates, respectively?

118. Three men, A, B, and C, agree to do a certain piece of work for \$52·90; A and B calculate that they can do $\frac{3}{4}$ of the work; A and C calculate that they can do $\frac{9}{10}$ of the work; and B and C $\frac{13}{20}$. They are to be paid proportionately, to these estimates. How much should each receive?

119. The stock of a certain bank is divided, into 32 shares, and is owned equally by eight persons, A, B, C, D, &c. A sells 3 of his shares to a ninth person, and B sells 2 of his shares to the *Company*. What proportion of the whole stock does A and B respectively still own?

120. A boy, being asked how many eggs he was carrying to market, replied, I do not know; but father said, if I had 1 dozen more, and should multiply this number by 2, and add to the product 2 dozen; and then sell them all, at $12\frac{1}{2}$ cents a dozen, I would receive for them \$1·50. How many eggs had he?

121. A farmer, being asked how many sheep he had, replied. that he had them in four different fields; and that $\frac{3}{4}$ of the number in the second field equalled $\frac{2}{3}$ of the number in the first; $\frac{2}{3}$ of the number in the second, equalled $\frac{3}{4}$ of the number in the third; and $\frac{2}{3}$ of the number in the third, equalled $\frac{4}{5}$ of the number in the fourth. How many sheep in each field, providing there are 64 more sheep in the third field than in the fourth, and how many in all?

122. A boy, having some oranges, sold to one person $\frac{1}{3}$ of all he had and 10 oranges more; to another, $\frac{1}{5}$ of the remainder and 10 more; to a third, $\frac{3}{10}$ of what then remained and 7 more; to a fourth, $\frac{2}{5}$ of what then remained and 2 more; to a fifth, $\frac{2}{5}$ of what still remained and 10 more; and to the sixth, the remainder. How many oranges had he at first, and how many did he sell to each individual, providing the fifth bought 12 oranges more than the sixth?

123. The interest of A's, B's, and C's fortune for nine years and 4 months, at 3 per cent., is \$30380. What is the fortune of each, providing $\frac{3}{4}$ of A's fortune equals $\frac{2}{3}$ of B's, and $\frac{3}{4}$ of B's equals $\frac{2}{3}$ of C's?

124. There is a rectangular box, 16 feet long, 4 feet wide, and 3 feet deep. What must be the length and width of another rectangular box of the same depth, that shall contain 5625 solid feet, providing its length and width are in the same proportion?

125. A man at his death, having a daughter in France, and a son in Russia, willed, if the daughter returned, and not the son, that the widow should have $\frac{3}{5}$ of the estate; and if the son returned, and not the daughter, that the widow should receive $\frac{2}{5}$ of the estate. They both returned. How much, according to the will, should each receive, providing the estate amounted to \$7600?

126. A, B, and C agree to do a certain piece of work for \$87·87; A and B can do the work in $6\frac{2}{3}$ days; B and C in 12 days; and A and C in 10 days. How much should each receive, according to the above estimates?

127. What will be the dimensions of a rectangular box, which shall contain 4037250 solid inches; the length, breadth, and depth being proportional to the numbers 7, 3, and 2?

128. A thief stole a horse from a farmer, B, and made off with it; 5 days after, B got intelligence of the direction the thief took, and followed him at the rate of 60 miles a day; and by so doing gained 20 per cent. on the thief. At what rate did the thief travel; how far must B ride before he overtakes him; and how many days will it require.

129. A drover being asked how many animals he had, replied, that $\frac{2}{5}$ of the number were sheep; $\frac{3}{5}$ of the remainder were hogs; and what then remained were calves; and that, if he should sell the sheep at \$$2\frac{1}{2}$ a head; his hogs at \$$3\frac{1}{2}$; and his calves at \$5 a head, he should receive \$519, which was \$119 more than they cost. How many sheep, hogs, and calves had he, respectively?

130. If 14 oxen eat 2 acres of grass in 3 weeks, and 16 oxen eat 6 acres in 9 weeks, how many oxen would eat 24 acres in 6 weeks; the grass being at first equal on every acre, and growing uniformly?

131. If 8 oxen eat 2 acres of grass in 3 weeks; and 15 oxen eat 5 acres in 6 weeks; for how many weeks can 15 oxen graze on 6 acres, the grass growing uniformly?

132. If 3 acres of grass, together with what grew on the 3 acres while they were grazing, keep 13 oxen 9 weeks, and in like manner, 4 acres keep 20 oxen 6 weeks, how many acres will be required to keep 36 oxen 4 weeks?

133. A general drew up his regiment in the form of a square and had 94 men remaining; soon after a detachment of 485 men more joined him, whereby he was enabled to increase the side of the square by 3 men. How many soldiers had he at first?

134 A market-woman carried some butter, strawberries

and eggs to market; she sold her butter, at 25 cents a pound; her strawberries at 20 cents a quart; and her eggs, at 15 cents a dozen; the whole amounted to $7·65. The number of pounds of butter equalled the number of dozens of eggs increased by the number of quarts of strawberries; and the number of pounds of butter increased by the number of quarts of strawberries, or the number of dozens of eggs, would equal 3 times as much as the remaining number. What was the quantity of each article?

135. A, B, C, and D agree to a certain piece of work, for $945; A, B, and C can perform the work in 84 days; A, B, and D, in 72 days; A, C, and D, in 63 days; and B, C, and D, in 56 days. How much money should each receive, providing they all work until the work is complete?

136. A, B, C, and D play cards on this condition: that he who loses shall give to all the others as much as they already have. First A lost, then B, then C, and then D. When they began to play they had $162, $82, $42, and $22, respectively; how much had each at the end of the fourth game? Suppose, when they had all lost in turn, that each had the same sum of money $96; how much had each when they commenced to play?

137. For three successive years a merchant, annually, contributed $150 for charitable purposes, and added yearly to that part of his capital not thus expended, a sum equal its $\frac{1}{3}$. At the end of the third year his original capital was doubled. What was his capital?

138. There is an island $26\frac{2}{3}$ miles in circumference, and three men A, B, and C, start from the same point, and travel in the same direction around it; A goes $2\frac{1}{2}$ miles an hour; B goes $8\frac{1}{3}$ miles an hour; and C goes $9\frac{2}{7}$ miles an hour. In what time will they all first be together; and when will they all be together at the place from which they started?

139. Three carpenters, A, B, and C, receive $26 for a certain amount of labor;—$\frac{4}{9}$ of the number of days B labored equaled $\frac{2}{3}$ of the number of days A labored, and $\frac{4}{5}$ of the number of days C labored equaled $\frac{2}{3}$ of the num-

ber of days B labored; and A labored as many days as C, lacking 5. How many days did each work, and how much did each receive a day, providing $\frac{1}{2}$ of A's daily wages equaled $\frac{2}{3}$ of B's, and $\frac{2}{3}$ of C's equaled $\frac{4}{9}$ of B's?

140. A and B paid $90 for 12 acres of pasture for 8 weeks, with an understanding that B should have the grass that was then on the field; and A, what grew during the time they were grazing. How many oxen according to the above understanding can each turn into the pasture, and how much should each pay, providing 4 acres of pasture, together with what grew during the time they were grazing, will keep 12 oxen six weeks; and in a similar manner, 5 acres will keep 35 oxen 2 weeks?

141. A gentleman has in one bank a certain number of 20, 15, and 10 dollar bills; in another a certain number of 5, and $2\frac{1}{2}$ dollar gold coins. The number of bills and coins in both banks equal 3224. How many of each has he, providing $\frac{3}{4}$ of the number of 20 dollar bills equal $\frac{2}{3}$ of the number of 15 dollar bills, $\frac{3}{4}$ of the number of 15 dollar bills equal $\frac{2}{3}$ of the number of 10 dollar bills, and $\frac{3}{4}$ of the number of 5 dollar gold coins are 48 more than $\frac{2}{3}$ of the number of $2\frac{1}{2}$ dollar coins; also, that $\frac{5}{7}$ of the number of bills equal $\frac{5}{6}$ of the number of coins; and what amount of money has he in both banks?

142. Divide a bar of lead weighing 40 pounds into four pieces, with which (and a pair of scales) any number of pounds from 1 to 40 may be weighed.

143. Find the least possible whole number which being divided by 28, shall leave 19 for a remainder; and being divided by 19, shall leave 15 for a remainder; and being divided by 15, shall leave 11 for a remainder?

THE END.

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